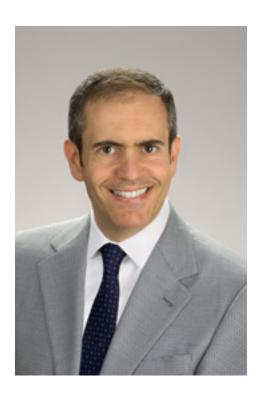


### **Our Presenter**



**Dr. Greg Glockner**Director of Engineering, Gurobi Optimization, Inc.



# Parallel & Distributed Optimization



### Terminology for this presentation

### Parallel computation

- One computer
  - Multiple processor cores
  - 1 or more processor sockets
- Part of Gurobi throughout our history
  - MIP branch-and-cut
  - Barrier for LP, QP and SOCP
  - Concurrent optimization

### Distributed computation

- Multiple computers, linked via a network
- Relatively new feature
- Each independent computer can do parallel computation!

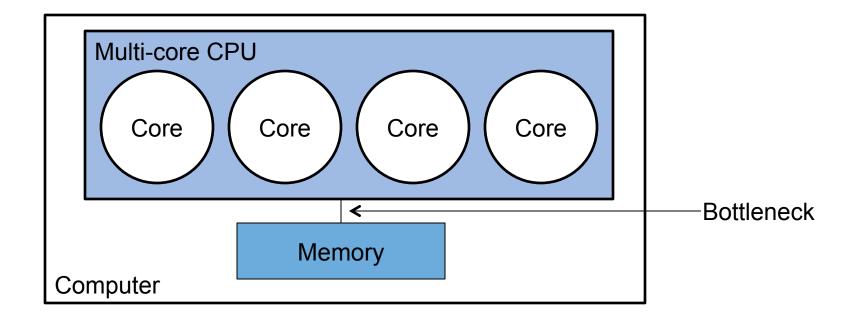


### Parallel algorithms and hardware

- Parallel algorithms must be designed around hardware
  - What work should be done in parallel
  - How much communication is required
  - How long will communication take
- Goal: Make best use of available processor cores

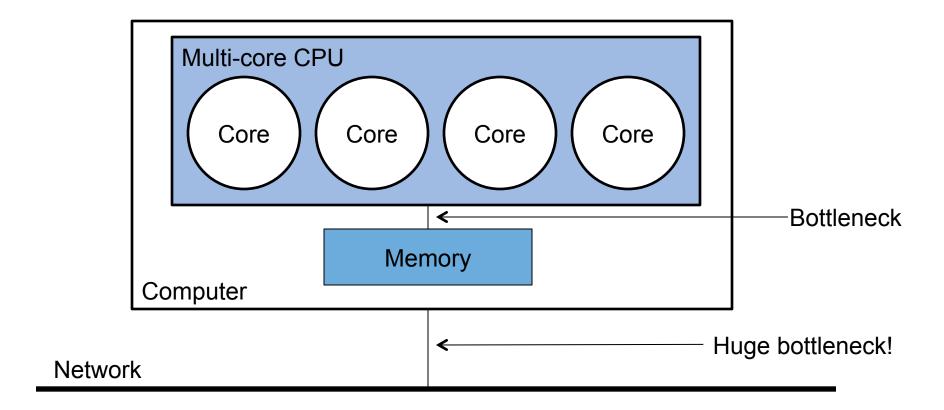


### **Multi-Core Hardware**



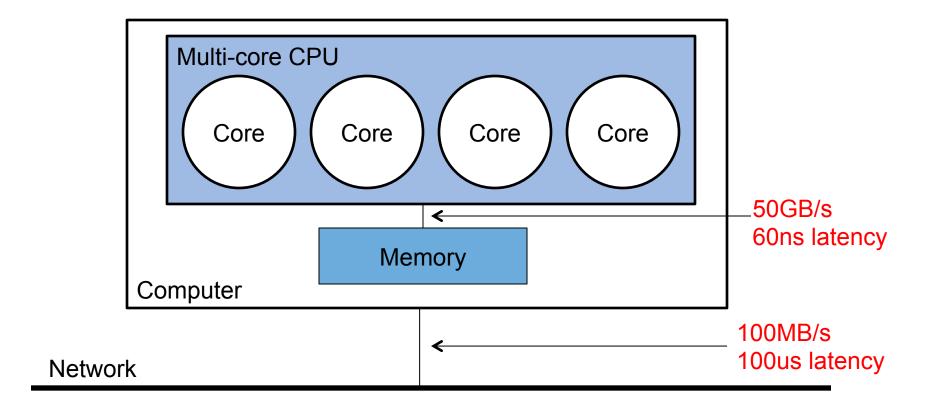


### **Distributed Computing**





### **How Slow Is Communication?**



- Network is ~1000X slower than memory
  - Faster on a supercomputer, but still relatively slow



### Distributed Algorithms in Gurobi 6.0

- 3 distributed algorithms in version 6.0
  - Distributed tuning
  - Distributed concurrent
    - LP (new in 6.0)
    - MIP
  - Distributed MIP (new in 6.0)



### **Distributed Tuning**

- Tuning:
  - MIP has lots of parameters
  - Tuning performs test runs to find better settings
- Independent solves are obvious candidate for parallelism
- Distributed tuning a clear win during model development
  - 10X faster on 10 machines
- Hard to go back once you have tried it



# **Concurrent Optimization**



### **Concurrent Optimization**

- Run different algorithms/strategies on different machines/cores
  - First one that finishes wins
- Nearly ideal for distributed optimization
  - Communication:
    - Send model to each machine
    - Winner sends solution back
- Concurrent LP:
  - Different algorithms:
    - Primal simplex/dual simplex/barrier
- Concurrent MIP:
  - Different strategies
  - Default: vary the seed used to break ties
- Easy to customize via concurrent environments



### **MIPLIB 2010 Testset**

- MIPLIB 2010 test set...
  - Set of 361 mixed-integer programming models
  - Collected by academic/industrial committee
- MIPLIB 2010 benchmark test set...
  - Subset of the full set 87 of the 361 models
    - Those that were solvable by 2010 codes
    - (Solvable set now includes 206 of the 361 models)

#### Notes:

- Definitely not intended as a high-performance computing test set
  - More than 2/3 solve in less than 100s
  - 8 models solve at the root node
  - ~1/3 solve in fewer than 1000 nodes



### **Distributed Concurrent MIP**

- Results on MIPLIB benchmark set (>1.00X means concurrent MIP is faster):
  - 4 machines vs 1 machine:

Runtime	Wins	Losses	Speedup
>1s	38	20	1.26X
>100s	17	3	1.50X

16 machines vs 1 machine:

Runtime	Wins	Losses	Speedup
>1s	54	19	1.40X
>100s	26	1	2.00X



### **Customizing Concurrent**

- Easy to choose your own settings:
  - Example 2 concurrent MIP solves:
    - Aggressive cuts on one machine
    - Aggressive heuristics on second machine
    - Java example

```
GRBEnv env0 = model.getConcurrentEnv(0);
GRBEnv env1 = model.getConcurrentEnv(1);
env0.set(GRB.IntParam.Cuts, 2);
env1.set(GRB.DoubleParam.Heuristics, 0.2);
model.optimize();
model.discardConcurrentEnvs();
```

Also supported in C++, .NET, Python and C

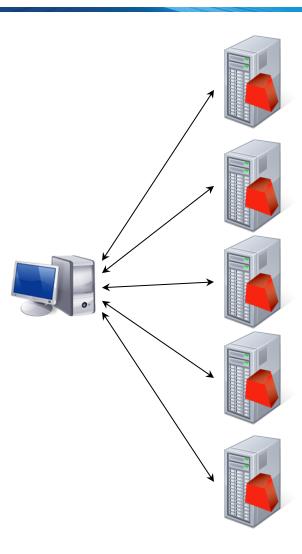


# **Distributed MIP**



### **Distributed MIP Architecture**

- Manager-worker paradigm
- Manager
  - Send model to all workers
  - Track dual bound and worker node counts
  - Rebalance search tree to put useful load on all workers
  - Distribute feasible solutions
- Workers
  - Solve MIP nodes
  - Report status and feasible solutions
- Synchronized deterministically





### **Distributed MIP Phases**

- Racing ramp-up phase
  - Distributed concurrent MIP
    - Solve same problem individually on each worker, using different parameter settings
    - Stop when problem is solved or "enough" nodes are explored
    - Choose a "winner" worker that made the most progress
- Main phase
  - Discard all worker trees except the winner's
  - Collect active nodes from winner, distribute them among now idle workers
  - Periodically synchronize to rebalance load



### **Bad Cases for Distributed MIP**

- Easy problems
  - Why bother with heavy machinery?
- Small search trees
  - Nothing to gain from parallelism
- Unbalanced search trees
  - Most nodes sent to workers will be solved immediately and worker will become idle again

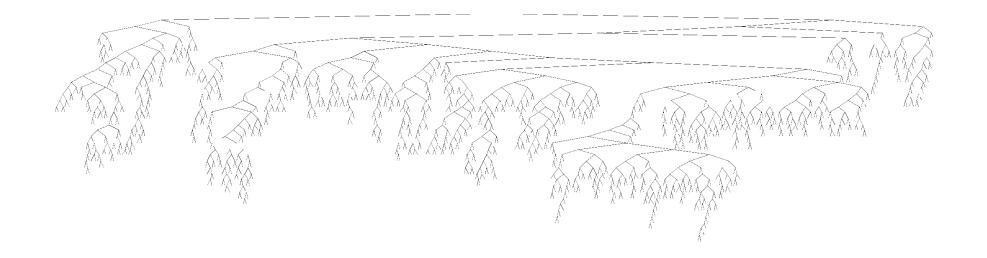
"neos3" solved with SIP (predecessor of SCIP)

Achterberg, Koch, Martin: "Branching Rules Revisited" (2004)



### **Good Cases for Distributed MIP**

- Large search trees
- Well-balanced search trees
  - Many nodes in frontier lead to large sub-trees



"vpm2" solved with SIP (predecessor of SCIP)

Achterberg, Koch, Martin: "Branching Rules Revisited" (2004)



# **Performance**



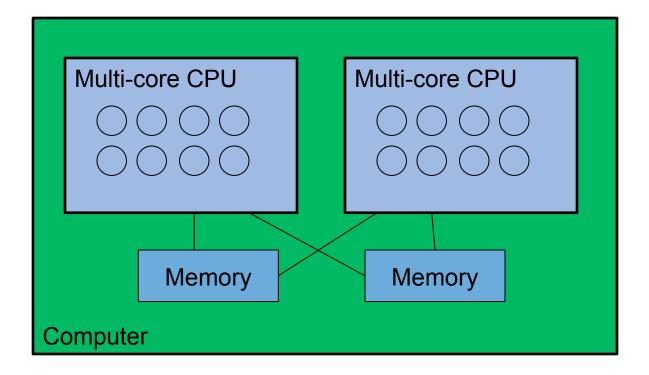
### **Three Views of 16 Cores**

- Consider three different tests, all using 16 cores:
  - On a 16-core machine:
    - Run the standard parallel code on all 16 cores
    - Run the distributed code on four 4-core subsets
  - On four 4-way machines:
    - Run the distributed code
- Which gives the best results?



### **Parallel MIP on 1 Machine**

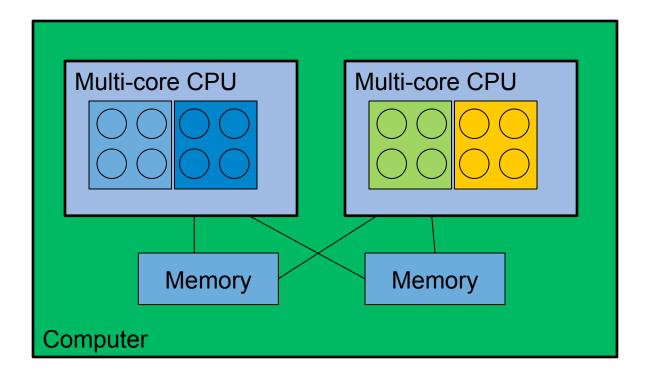
Use one 16-core machine:





### **Distributed MIP on 1 machine**

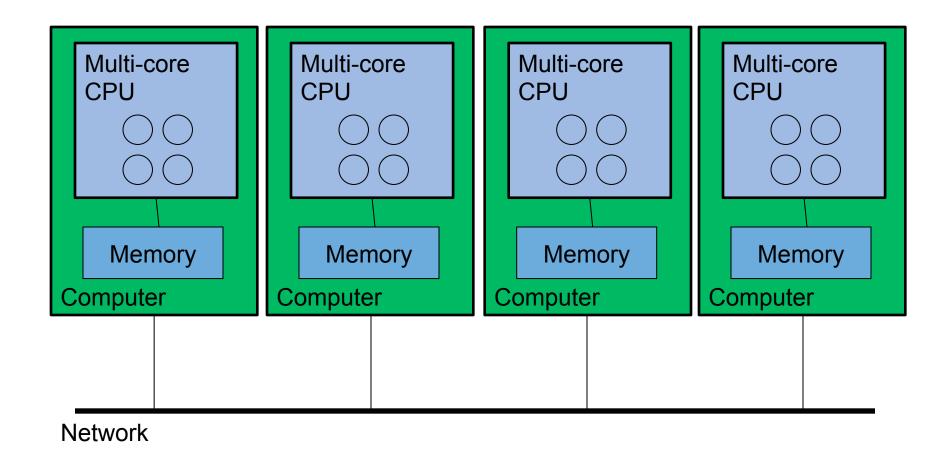
▶ Treat one 16-core machine as four 4-core machines:





### **Distributed MIP on 4 machines**

Use four 4-core machines





### **Performance Results**

Using one 16-core machine (MIPLIB 2010, baseline is 4-core run on the same machine)...

Config	>1s	>100s
One 16-core	1.57X	2.00X
Four 4-core	1.26X	1.82X

- Better to run one-machine algorithm on 16 cores than treat the machine as four 4-core machines
  - Degradation isn't large, though



### **Performance Results**

Comparing one 16-core machine against four 4-core machines (MIPLIB 2010, baseline is single-machine, 4-core run)...

Config	>1s	>100s
One 16-core machine	1.57X	2.00X
Four 4-core machines	1.43X	2.09X

- Given a choice...
  - Comparable mean speedups
  - Other factors...
    - Cost: four 4-core machines are much cheaper
    - · Admin: more work to admin 4 machines



### **Distributed Algorithms in 6.0**

- MIPLIB 2010 benchmark set
  - Intel Xeon E3-1240v3 (4-core) CPU
  - Compare against 'standard' code on 1 machine

Madaina		>1s		>100s		
Machines	Wins	Losses	Speedup	Wins	Losses	Speedup
2	40	16	1.14X	20	7	1.27X
4	50	17	1.43X	25	2	2.09X
8	53	19	1.53X	25	2	2.87X
16	52	25	1.58X	25	3	3.15X



### **Some Big Wins**

- Model seymour
  - Hard set covering model from MIPLIB 2010
  - 4944 constraints, 1372 (binary) variables, 33K non-zeroes

Machines	Nodes	Time (s)	Speedup
1	476,642	9,267s	_
16	1,314,062	1,015s	9.1X
32	1,321,048	633s	14.6X



### **Distributed Concurrent Versus Distributed MIP**

- MIPLIB 2010 benchmark set (versus 1 machine run):
  - · >1s

Machines	Concurrent	Distributed
4	1.26X	1.43X
16	1.40X	1.58X

• >100s

Machines	Concurrent	Distributed
4	1.50X	2.09X
16	2.00X	3.15X



### **Gurobi Distributed MIP**

- Makes huge improvements in performance possible
- Mean performance improvements are significant but not huge
  - Some models get big speedups, but many get none
  - Much better than distributed concurrent
  - As effective as adding more cores to one box
- Effectively exploiting parallelism remains:
  - A difficult problem
  - A focus at Gurobi



# **Mechanics**



### **Gurobi Remote Services**

- Install Gurobi Remote Services on worker machines
  - No Gurobi license required on workers
  - Machine listens for Distributed Worker requests
- Set a few parameters on manager
  - ConcurrentJobs=4
  - WorkerPool=machine1, machine2, machine3, machine4
  - No other code changes required
- Manager must be licensed to use distributed algorithms
  - Gurobi Distributed Add-On
    - Enables up to 100 workers



### **Integral Part of Product**

- Built on top of Gurobi Compute Server
  - Only 1500 lines of C code specific to concurrent/distributed MIP
- Built into the product
  - No special binaries involved
- Bottom line:
  - Changes to MIP solver automatically apply to distributed code too
    - Performance gains in regular MIP also benefit distributed MIP
  - Distributed MIP will evolve with regular MIP



### Footnote: GPGPU computing

- GPGPU: General-purpose computing on Graphics Processing Units
  - Massively parallel for simple computation
  - Heavily marketed for parallel tasks
- Currently, GPUs are not well-suited for solvers like Gurobi
  - For LP, sparse linear algebra does not parallelize to hundreds of GPUs
  - For MIP, each tree node requires very different calculations, but GPU SIMD computations are designed for identical calculations on different data
- General-purpose CPUs continue to add parallel cores, which benefit Gurobi Optimizer



### **Distributed Optimization Licensing**

#### Commercial

- Not included must purchase the distributed option
- Ask your sales representative for benchmarks or pricing

#### Academic

- Named-user: not included in licenses from Gurobi website
- Site license: not currently supported
- If interested, your network administrator must contact Gurobi support to request a single-machine, distributed license

#### Cloud

- Distributed optimization will be prepackaged in the new release of Gurobi Cloud, later in 2015
- All licenses include parallel optimization on a single computer



