GotSport

Sports Scheduling with Gurobi

Concept & Use Case

GUROBI OPTIMIZATION

Walter Sebastian Gisler
walter@gotpro.com

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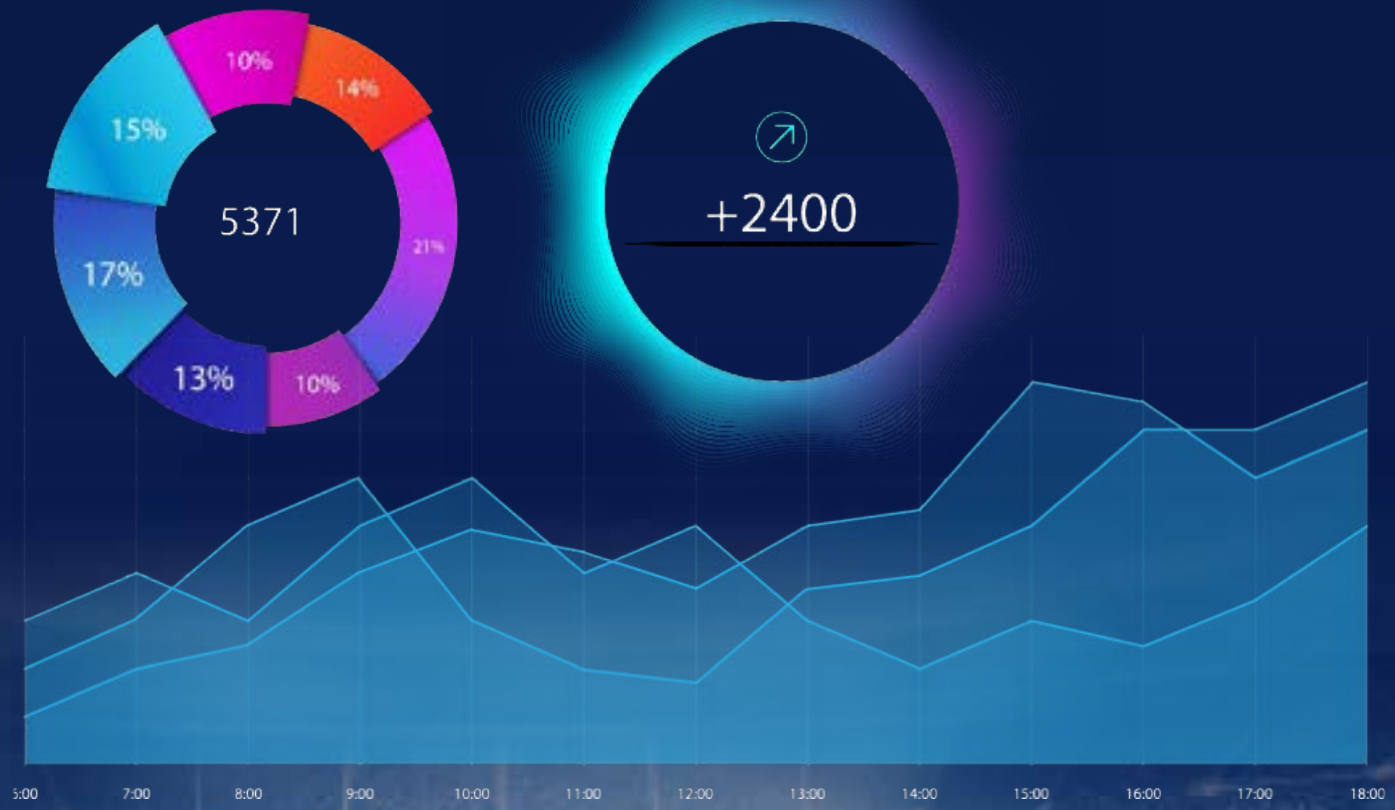
Agenda

- A bit of background
- Optimization opportunities in sports
- Sports Scheduling Basics
- Mathematical Modelling approach
- Challenges
- Case Study: CBF



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- Leader in Youth Sports since 1996
- 4.2M registered players in the USA
- Working with > 60 professional sports leagues across the globe
- Soccer, Rugby, Basketball, Volleyball, Cricket, Lacrosse,
- Handball, American Football, Hockey, etc.
    - GotSport Pro
    - GotSport
    - GotSport Analytics
- Our 4th year as a Gurobi client

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Team grouping
And seeding
Optimal seat
assignment
in a stadium
Referee
Scheduling
Match scheduling
Pro
Youth
Broadcast Scheduling
Event Scheduling
Competition Format
Organization
Pricing
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Sports Scheduling Basics

The importance of a good schedule

Sports Scheduling Basics

A seemingly simple problem…

- N-1 match days
- Every team plays each opponent once

- Task:
- Plan a matchup between each pair of teams and assign a match day to each matchup

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Sports Scheduling Basics

Subject to...

- One matchup per team, per weekend
- For each team, half of the matches (+/-1) should be played at home, and the other half away
- Etc.

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A:B	B:C	C:D	D:E	E:F	
A:C	B:D	C:E	D:F		
A:D	B:E	C:F			
A:E	B:F				
A:F					

?

	1	2	3	4	5
A	F	B	C	D	E
B	E	A	F	C	D
C	D	E	A	B	F
D	C	F	E	A	B
E	B	C	D	F	A
F	A	D	B	E	C

Sports Scheduling Basics

A MIP formulation

$$\min \sum_{i \in T} \sum_{j \in T \setminus \{i\}} \sum_{p \in P} c_{i,j,p} x_{i,j,p}$$

$$\begin{aligned}
\text{s.t.} \sum_{p \in P} (x_{i,j,p} + x_{j,i,p}) &= 1 && \forall\, i, j \in T, i < j \\
\sum_{j \in T \setminus \{i\}} (x_{i,j,p} + x_{j,i,p}) &= 1 && \forall\, i \in T, p \in P \\
x_{i,j,p} &\in \{0,1\} && \forall\, i, j \in T, i \neq j, p \in P
\end{aligned}$$

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Sports Scheduling Basics

Isn’t this simple?

- Assuming, we have 20 teams….
- 20x19x19 = 7220 binary variables
- 20x19x2 = 760 constraints

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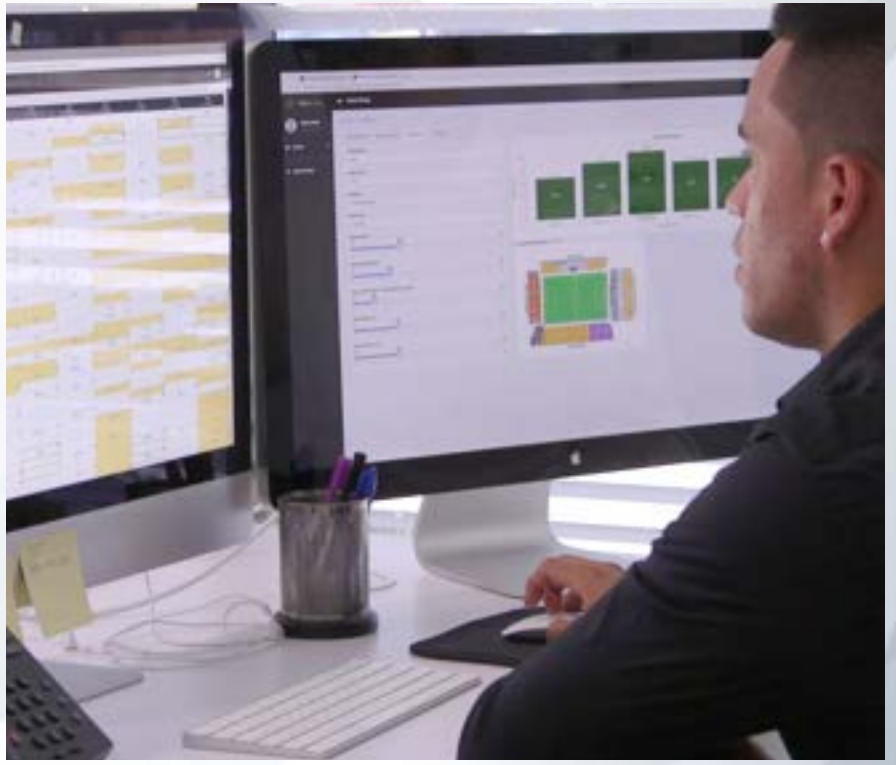
Yes, but...

- Huge number of possible solutions
- Not trivial at all to find these solutions by hand and finding a solution with certain additional characteristics is very hard
- NP-complete problem



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	1	2	3	4	5
A	F	B	C	D	E
B	E	A	F	C	D
C	D	E	A	B	F
D	C	F	E	A	B
E	B	C	D	F	A
F	A	D	B	E	C

	1	2	3	4	5
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D	C	E	F	A	B
E	B	D	C	F	A
F	A	B	D	E	C

Idea 1: let's just change home opponent

- A plays 4 consecutive games at home
- C plays 3 consecutive games away

Idea 2: let's swap round 2 and 3

- All of a sudden there are a lot of cases, where a team plays two consecutive home or two consecutive away matches

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Sports Scheduling Basics

Growing Complexity and Challenges

Conflicting Restrictions and Rules

Police Requests

Venue Availability

Schedule Fairness

Minimize Travel

Maximize TV revenue

Event Overlap

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Sports Scheduling Basics

Advantages of MIP solvers

- Conflicting constraints can be found more easily
- It is easy to adjust constraints and add more rules, which could be difficult with other techniques
- Solution quality is consistently high
- Can be easily combined with other techniques,
- e.g. local search
- Cost efficient: faster development, short turnaround time

Sports Scheduling Basics

Tricks to solve hard problems

- Decomposition of the problem:
- First assign H/A, then find a match assignment
- Break the problem into different parts (e.g. first solve the first 10 rounds, then the rest)
- Not always possible, Impact on feasibility needs to be analyzed carefully
- Large Neighborhood Search, Fix-And-Relax Heuristic
- Very powerful if an initial (near-)feasible solution can be found easily

Case Study: CBF

A particularly hard optimization challenge

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CBF Serie A

- 20 Teams
- Double Round Robin Tournament
- Relatively few team specific restrictions
- Strong focus on fairness
- Consecutive home/ away games (HH or AA) need to be minimized and balanced out across teams
- No tolerance for HHAHH or AAHAA sequences
- Spacing of matches against strong teams is very important

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CBF Serie A

- Broadcasting requests:
- Derbies (classicos) should be spread out as much as possible
- Some travel concerns
- Equal home balance for all teams across midweek dates
- Cross pairing constraints

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Solving this…

- If this is modelled as a single model and passed to Gurobi, no solution would be found, even after 30 days
- A slightly better way to model it, is to relax some of the hard constraints
- For example, instead of strictly forbidding three consecutive Home games (HHH), it can be added as a soft constraint, with a high penalty
- This approach would find a solution after a few days, but it would still not find a high-quality solution after 30 days

Clearly, this isn't the way to go

- Solutions need to be found faster (hours instead of weeks)
- It is essential for clients to review multiple solutions and not just present one final "most optimal solution"
- As with all optimization projects, it is hard to capture all requirements
- A client might prefer a solution with a "worse" score, because of some characteristics he didn't mention
- With multi-objective optimization problems, weighing the different objectives is always hard

Decomposition approach

- Two approaches from literature (see: Dirk Briskorn):
- First schedule matchups, then decide on who is home for each matchup
- For each team and matchday, decide who plays home, then find matches that satisfy these
- additional restrictions
- (”First HAP, then schedule”)

First HAP, then schedule

- Not every HAP can be used to generate a feasible schedule (in fact, only a tiny fraction)
- What are the characteristics of a “feasible HAP”?
- Big challenge: how to find HAPs for which a corresponding schedule exists?
- Enumeration?
- MIP?
- Constraint programming?

Closing Remarks & Summary

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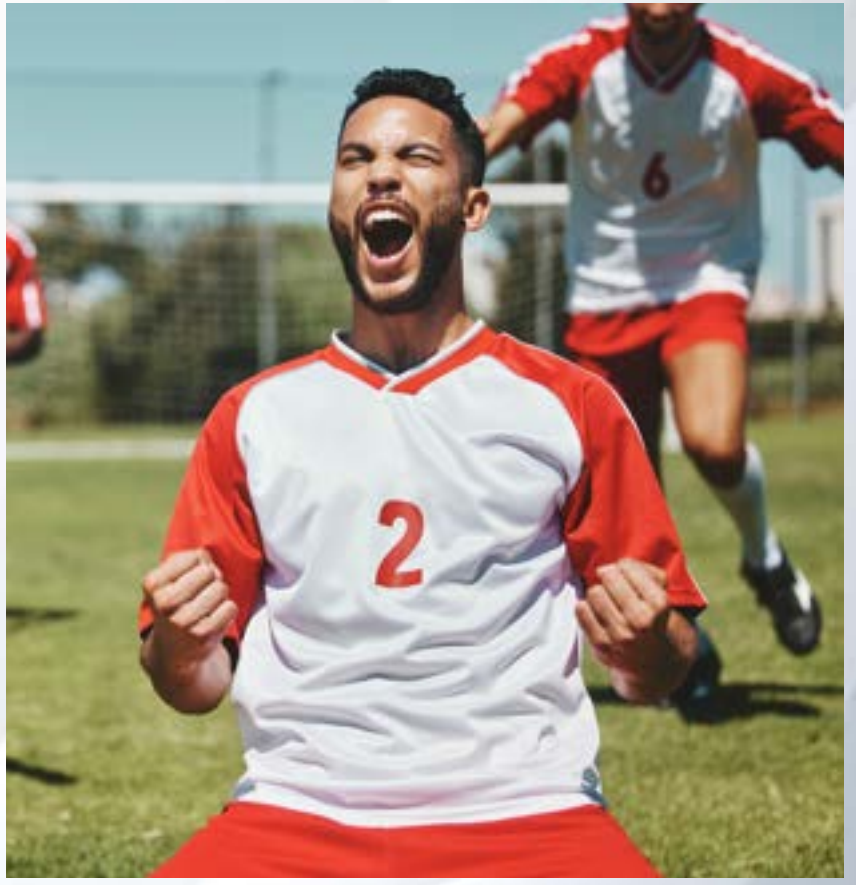
Summary

- Optimization can make a big impact in Sports
- Reduce costs
- Use resources more efficiently
- Create a more exciting and competitive season
- Increase revenues of leagues and clubs



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