Adapting Groundwater Abstraction Management Using Mathematical Optimization

Water Corporation, Western Australia

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Think climate change.
Be waterwise.

Who Are We?

Water Corporation

State Government Owned

WATER

WASTEWATER

DRAINAGE

4,000+
EMPLOYEES

1,000,000+
CONNECTIONS

Our Team – Advanced Analytics

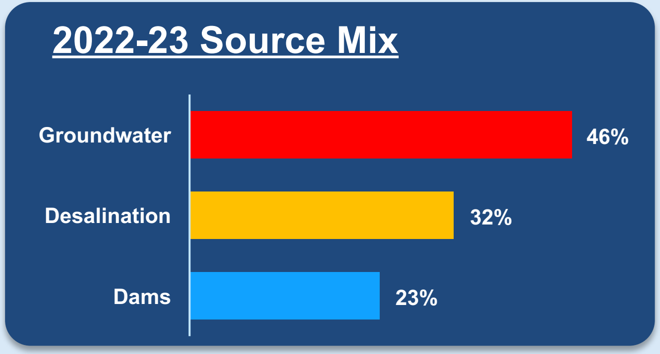
Perth Water Supply

The PWS Supplies ~ 300GL Annually to:

- Over 2 Million Customers
- Perth Metropolitan Area
- Export to Goldfields & Agricultural
- Some South West Regional Towns

Integrated System:

- 15 Dams
- 6 Groundwater treatment plants
  - 160 licenced artesian bores (wells)
- 2 Desalination plants

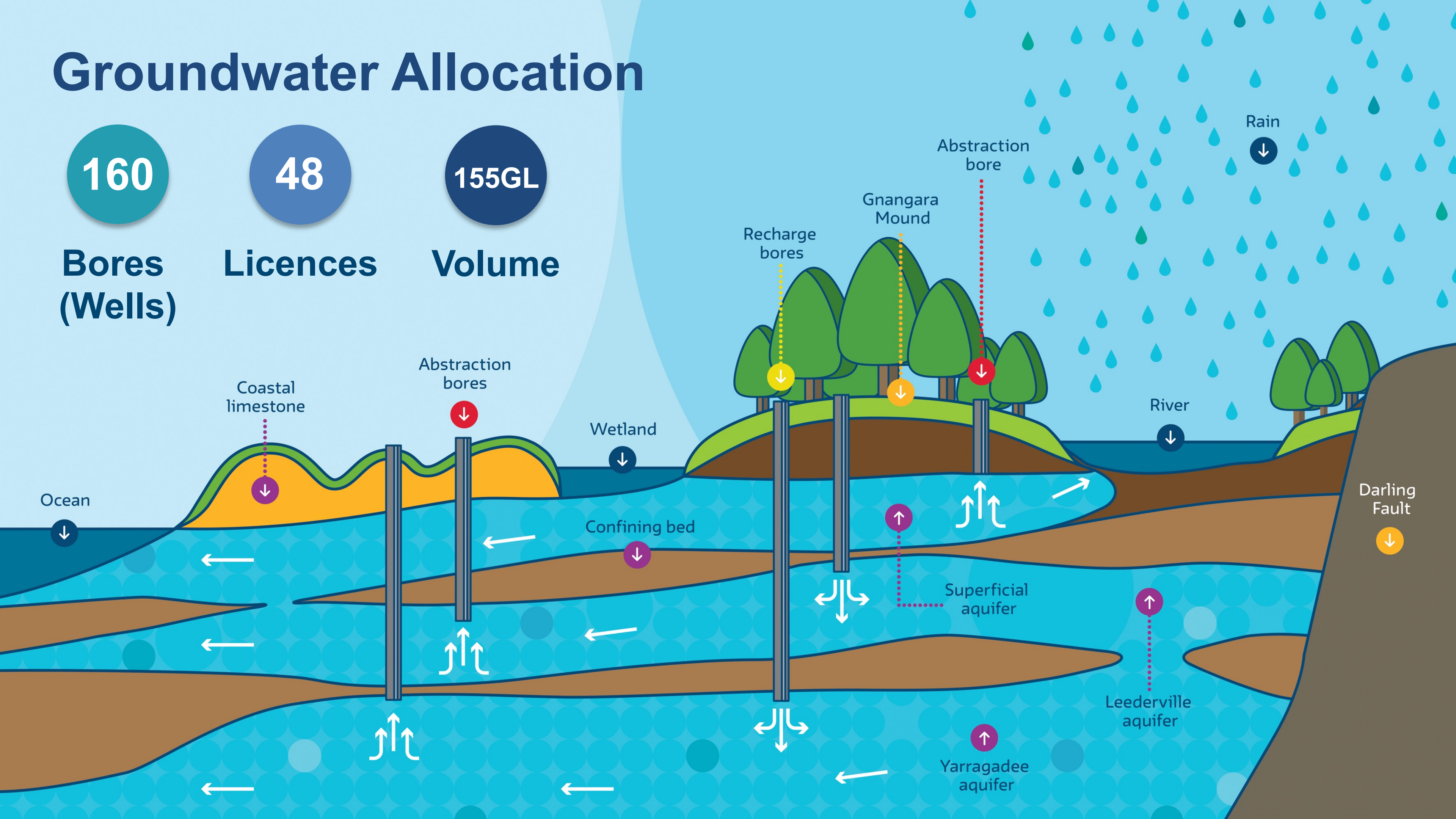


Groundwater Allocation

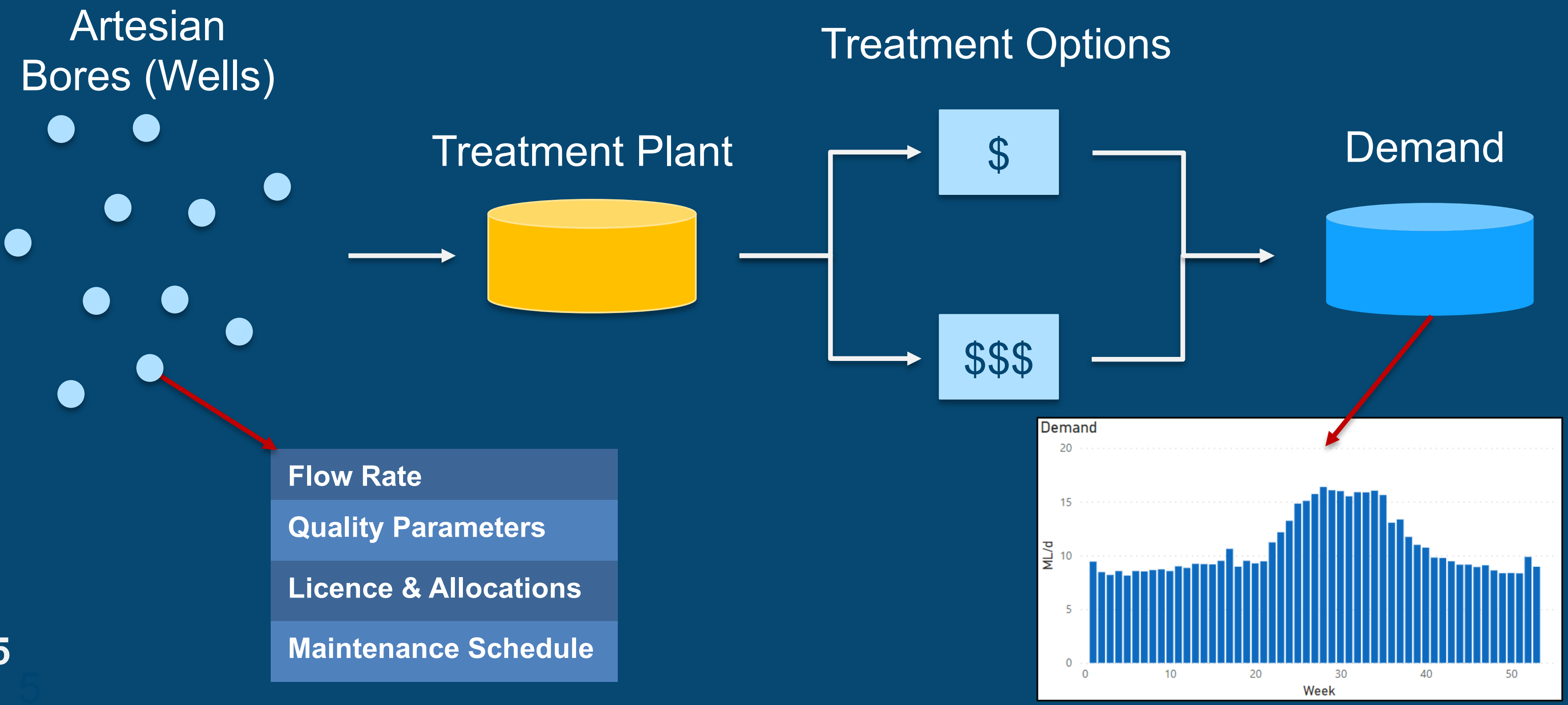
160
Bores (Wells)

48
Licences

155GL
Volume

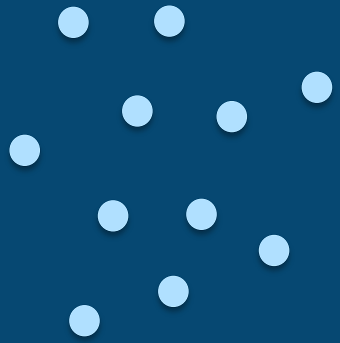


Groundwater Production



Groundwater Production

Artesian Bores (Wells)



Production Schedule

Season Week Starting borefield_id	13 21/09/2023 - 12 AM Ends on: 27/09/2023	14 28/09/2023 - 12 AM Ends on: 04/10/2023	15 05/10/2023 - 12 AM Ends on: 11/10/2023	16 12/10/2023 - 12 AM Ends on: 18/10/2023	17 19/10/2023 - 12 AM Ends on: 25/10/2023	18 26/10/2023 - 12 AM Ends on: 01/11/2023	19 02/11/2023 - 12 AM Ends on: 08/11/2023	20 09/11/2023 - 12 AM Ends on: 15/11/2023	21 16/11/2023 - 12 AM Ends on: 22/11/2023	22 23/11/2023 - 12 AM Ends on: 29/11/2023	23 30/11/2023 - 12 AM Ends on: 06/12/2023	07/12/ Ends o
⊟ Quinns												
⊟ *Superficial												
Q10	7.00	7.00	7.00	7.00	7.00	7.00	0.00	7.00	0.00	0.00	7.00	
Q20	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	
Q30	0.00	0.00	0.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	
Q40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
Q50	7.00	7.00	0.00	7.00	7.00	0.00	7.00	7.00	7.00	7.00	7.00	
Q60	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	
Q110	0.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	
Q120	7.00	7.00	7.00	7.00	0.00	7.00	7.00	7.00	7.00	7.00	7.00	
Q130	7.00	7.00	7.00	7.00	7.00	7.00	7.00	0.00	7.00	7.00	7.00	
Q140	0.00	0.00	7.00	7.00	7.00	7.00	0.00	7.00	7.00	7.00	7.00	
Q150	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	0.00	
Q170	7.00	7.00	7.00	7.00	7.00	7.00	0.00	0.00	0.00	0.00	0.00	
Q180	0.00	0.00	0.00	0.00	0.00	0.00	7.00	7.00	7.00	0.00	0.00	
Q190	0.00	0.00	0.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	
Q200	0.00	0.00	0.00	0.00	0.00	0.00	7.00	7.00	7.00	7.00	7.00	
⊟ Leederville												
Q35	0.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	
Q145	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	
Q155	0.00	0.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	
Q165	0.00	0.00	7.00	0.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	
Q205	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	0.00	7.00	7.00	
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WT40	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	
WT50	0.00	0.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	0.00	7.00	
WT60	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	
WT70	7.00	7.00	0.00	0.00	0.00	7.00	7.00	7.00	7.00	7.00	7.00	
WT80	0.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	0.00	7.00	7.00	
WT90	7.00	0.00	0.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	
⊟ Leederville												
WT5	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	

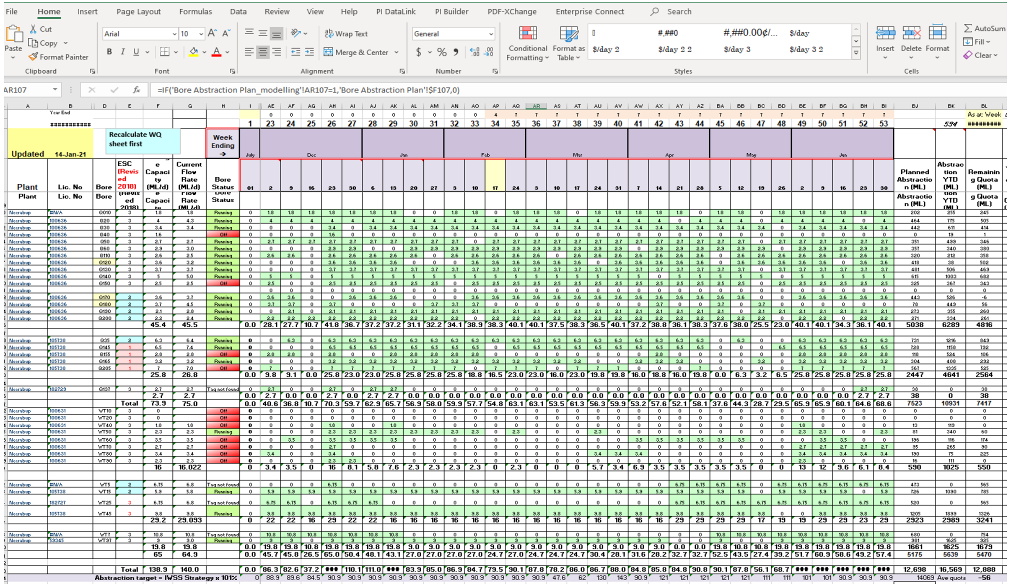
Think climate change.
Be waterwise.

WATER CORPORATION

Historical Approach

Basic strategy with Excel

- Run bores (wells) with **greatest remaining quota** to meet demand
- **Tweak** groundwater blend reactively to achieve treatable **quality** limits
- **Monitor** licences - transfer quota or shutdown bores
- Solving strategy is **greedy**



Functionality

- Manual & time intensive process
- Not responsive to dynamic nature of operations

Think climate change.
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The Impossible Problem

Data Analytics team proposed using Mathematical Optimisation.

- **Initially found it to be a mathematically impossible problem to solve.**
  - Over constrained
  - “Fluid” business rules were overlooked
- **Need to be flexible to achieve “robustness”**
  - Build in flexible constraints
  - To achieve an **acceptable** solution.

Optimisation Project

Goal:

- Produce an optimised groundwater blend such that the production volume is maximised, and chemical treatment costs are minimised.
  - For full 52 weeks of water year.
  - Whilst meeting all business rule requirements.

Modelling:

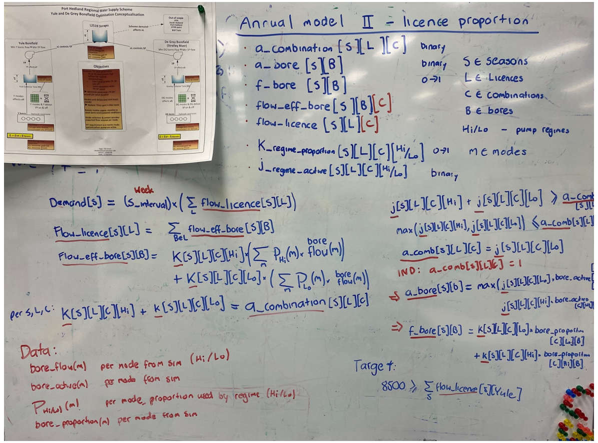
- Business rules into mathematical constraints.
  - Actions, decisions, limits need to be codified.
  - Apply reasonable assumptions and simplifications
- Uses mixed integer programming (MIP)

Stakeholder Engagement

- It was critical to regularly engage with the operational planner to understand the processes and then develop into a model.
- Identification of edge cases and exceptions for constraints was iterative.
- For expansion to regional implementations, meeting directly with staff on the ground identified further processes to model.

Model Development Process

- Receive plans, diagrams and documents relating to scheme of interest, and discuss with stakeholders.
- Identify objective and discrete rules for constraints.
- Prototype on-paper/whiteboard math model that describes the constraint. Check for edge cases to ensure robustness.
- Iterate with stakeholder for accuracy of modelled constraints
- Implement in code and inspect results.
- Feedback with stakeholder if results are logical.



Model Constraints

Fundamentals
State decision variables, transition, auxiliary relationships, time logic.

Bore (Well) event limit
To control the number of starts and stops per bore (well).

Partial week activation
To allow a bore to run for a fraction of a week.

Bore licence and allocation
Limits on the volume a bore (well) and group of bores under a licence can abstract annually.

Water quality
Alkalinity, iron, hardness, total dissolved solids (TDS), turbidity and UV.
Mass balance of raw blend per week.

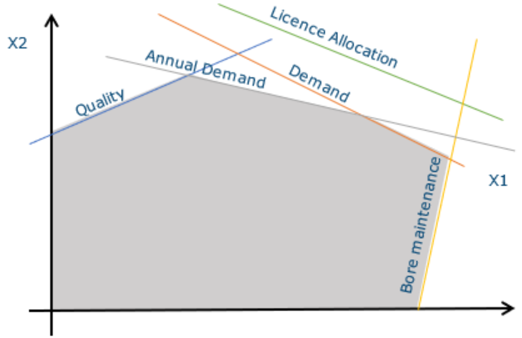
Bore (Well) maintenance and enforced use
To omit or make a bore activation mandatory in the solution schedule.

Water banking
Enable transfer of volume between weeks in the model (virtual storage reservoir).

Demand, weekly and annual limits
Upper and lower limits on the weekly volume, considering a penalty for being below target.
Lower bound on total annual volume abstracted, to eliminate unsatisfactory solutions.

Plant capacity
Upper and lower limits on the treatment plant volume capability.

…and more.



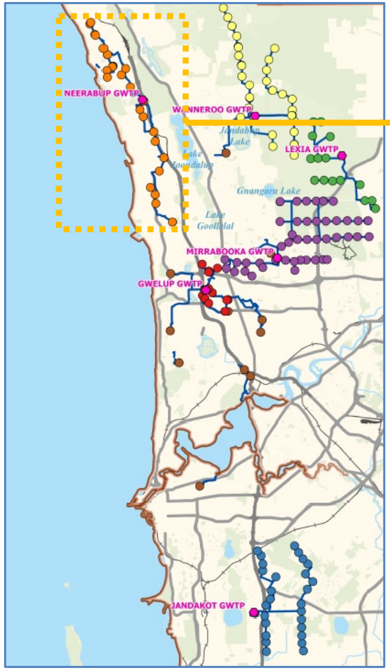
Introducing Non-Linearity

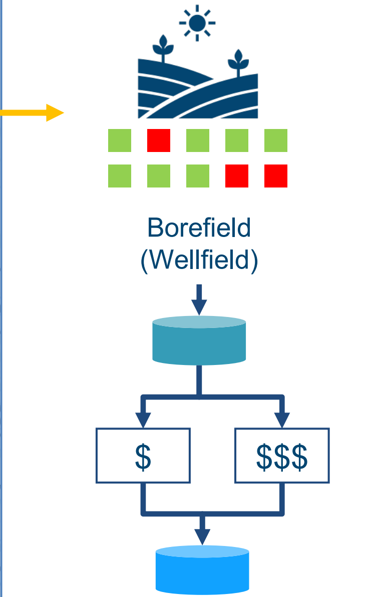
Optimise simultaneously:

- **The raw water blend,**
- **AND treatment stream proportion.**

- Choosing between treatment processes of different strength and cost.
  - Subject to numerous constraints.
  - This becomes a quadratic (i.e. **non-linear**) constraint we need to solve for.

- This enables the choice of a poor blend with the stronger treatment stream at higher proportions for part of the year, yielding significantly more flexibility and treatment capability.





Mathematical Model

Objective function:

$$Maximise \sum_{\substack{t \in T, \\ b \in B}} R_b\, a_{b,t} - \delta_t - \sum_{x \in X} k_{x,t} C_x$$

where R_b *is the bore (well) flow rate,* $a_{b,t}$ *whether a bore is active in each week,* δ_t *the demand deviation penalty,* $k_{x,t}$ *the volume per stream,* C_x *the cost per stream.*

Subject to:

$$a_{b,t}, b \in B\ (bores), t \in T (weeks), a_{b,t} \in \{0,1\}$$

$$f_{b,t}, \quad b \in B, \quad t \in T, \quad f_{b,t} \in [0,1]$$

$$\delta_t\ (continous), t \in T$$

$$k_{x,t}\ (continuous), x \in X\ (streams), t \in \mathrm{T}$$

$$\sum_{t \in T} v_{b,t} \leq E_b, \forall\, b \in B$$

$$f_{b,t} \leq 1, \forall\, b \in B, t \in T$$

$$\sum_{t \in T} f_{b,t} R_{b,l} + V_b^{consumed} \leq \sum_{t \in T} V_{b,l}$$

$$\sum_{t \in T} f_{b,t} R_b + V_b^{consumed} \leq V_b$$

$$\sum_{b \in B,} Q_{i,b} R_b\, f_{b,e,t} \leq Q_i^{max} \sum_{b \in B} R_b a_{b,e,t}$$

$$\sum_{b \in B} f_{b,,t} R_b \leq 0\ \ (maintenance)$$

$$\sum_{b \in B} f_{b,,t} R_b = 1\ (enforced)$$

$$\sum_{b \in B} R_b f_{b,t} + o_t \leq D_t + D_{upper}$$

$$\sum_{b \in B} R_b f_{b,t} + o_t \geq D_t - \delta_t$$

$$\sum_{\substack{b \in B \\ t \in T}} R_b f_{b,t} \geq D_{annual}$$

$$Lower \leq \sum_{b \in B} R_b\, f_{b,t} \leq Upper$$

$$\sum_{x \in X} p_{x,t} * (1 - P_{x,i}) * \sum_{b \in B} (Q_i R_b f_{b,t}) \leq Q_i^{max} \sum_{b \in B} R_b f_{b,t}$$

$$\sum_{b \in B} (Q_i R_b f_{b,t}) - \sum_{x \in X} (P_{x,i} * k_{x,t}) \leq Q_i^{max} \sum_{b \in B} R_b f_{b,t}$$

$$\sum_{x \in X} (R_b\, f_{b,t}) * p_{x,t} = k_{x,t}$$

$$\sum_{x \in X} p_{x,t} = 1$$

$$\sum_{x \in X} R_b\, f_{b,t} = \sum_{x \in X} k_{x,t}$$

For one treatment plant, the model has over 48,000 constraints and 78,000 variables.

Coding & Solving

Python

- Flexible tool for data processing.
- Automate processes such as input data validation.
- Management of constraints unique to schemes.

OSIsoft PI

- Collect operational data from time series historian into model to enable dynamic response to system changes.

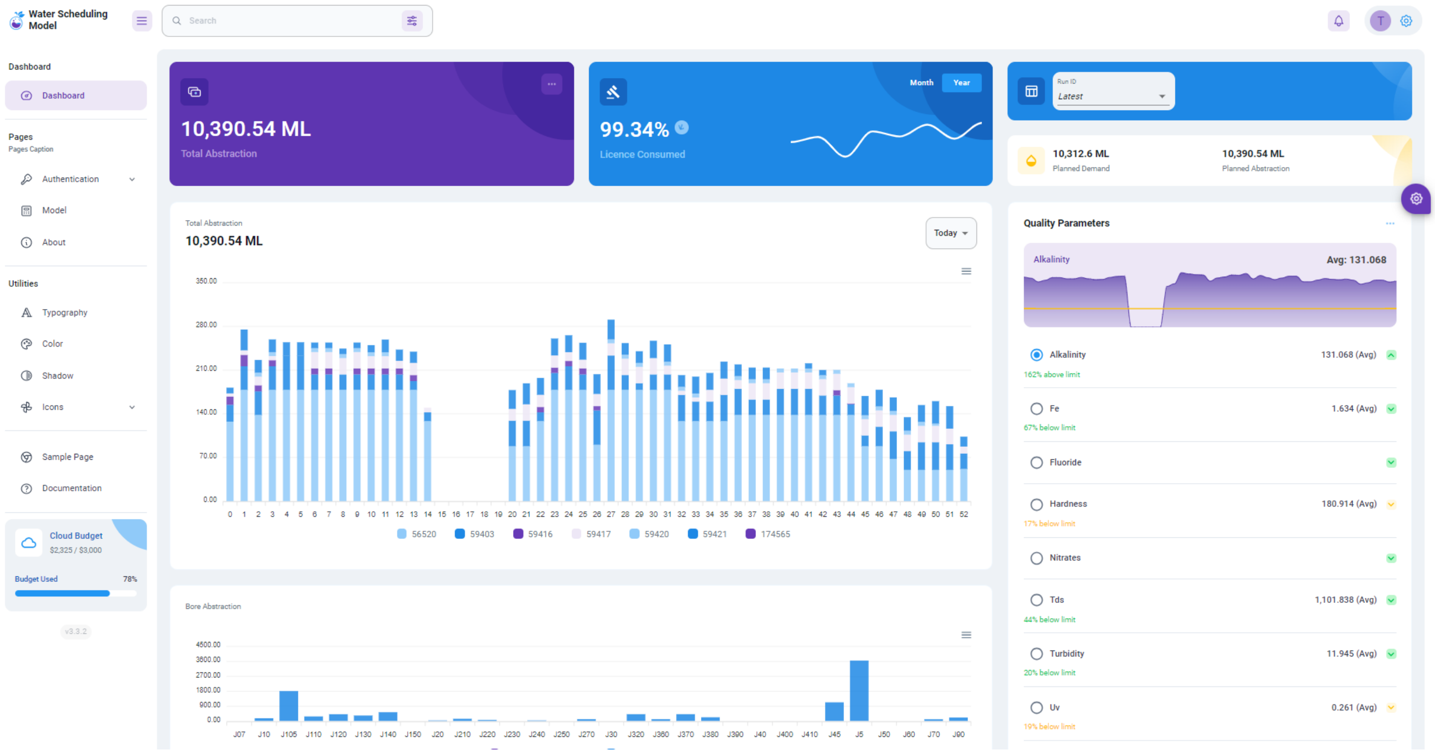
Gurobi

- Specialist mathematical optimisation solver controlled via Python API.
- Allows models to be solved with advanced techniques.

User Interface

React JS

Flask



Think climate change.
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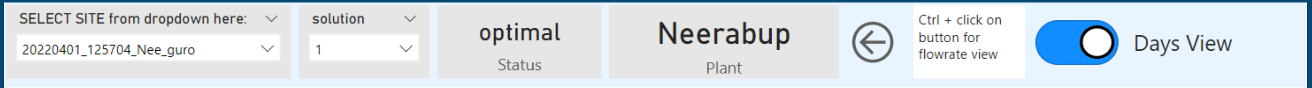
WATER CORPORATION

Visualisation – Power BI

Model Solution:

Bore (Well) Activation Schedule (days/week)

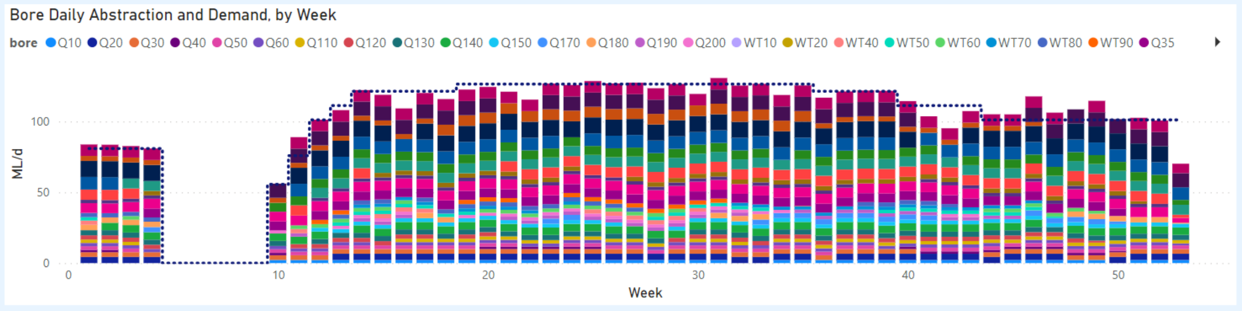


Season	40	41	42	43	44	45	46	47	48	49	50
Week Starting	31/03/2022 - 12 AM	07/04/2022 - 12 AM	14/04/2022 - 12 AM	21/04/2022 - 12 AM	28/04/2022 - 12 AM	05/05/2022 - 12 AM	12/05/2022 - 12 AM	19/05/2022 - 12 AM	26/05/2022 - 12 AM	02/06/2022 - 12 AM	09/06/2022 - 12 AM
borefield_id	Ends on: 6/04/2022	Ends on: 13/04/2022	Ends on: 20/04/2022	Ends on: 27/04/2022	Ends on: 4/05/2022	Ends on: 11/05/2022	Ends on: 18/05/2022	Ends on: 25/05/2022	Ends on: 1/06/2022	Ends on: 8/06/2022	Ends on: 15/06/2022
Quinns											
***Superficial**											
Q10	7.00	7.00	0.00	7.00	0.00	7.00	0.00	0.00	7.00	7.00	0.00
Q20	7.00	7.00	7.00	0.00	0.00	0.00	7.00	7.00	7.00	7.00	7.00
Q30	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
Q40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q50	7.00	7.00	7.00	7.00	7.00	0.00	7.00	7.00	7.00	7.00	7.00
Q60	7.00	7.00	0.00	7.00	7.00	7.00	7.00	7.00	7.00	0.00	7.00
Q110	7.00	7.00	7.00	7.00	0.00	7.00	0.00	7.00	7.00	7.00	7.00
Q120	7.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07
Q130	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
Q140	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	0.00
Q150	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	0.00	7.00
Q170	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.31
Q180	0.00	0.00	4.41	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q190	0.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	0.00	7.00	7.00
Q200	0.00	7.00	7.00	7.00	0.00	7.00	0.00	0.00	7.00	7.00	7.00
Leederville											
Q35	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
Q145	7.00	7.00	0.00	0.00	7.00	0.00	0.00	7.00	0.00	7.00	0.00
Q155	7.00	0.00	0.00	0.00	7.00	0.00	0.00	7.00	0.00	0.00	0.00
Q165	7.00	0.00	7.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q205	0.00	7.00	7.00	7.00	0.00	7.00	7.00	0.00	7.00	0.00	0.00
Yarragadee											
Q137	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
Whitfords											
***Superficial**											
WT10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WT20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WT40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.68	0.00
WT50	0.00	7.00	0.00	0.00	0.00	7.00	7.00	7.00	0.00	0.00	0.00
WT60	7.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WT70	0.00	0.00	0.00	3.43	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WT80	0.00	0.00	7.00	0.00	7.00	0.00	7.00	0.00	7.00	0.00	0.00
WT90	0.00	0.00	0.00	0.00	0.00	7.00	0.00	0.00	0.00	0.00	0.00
Leederville											
WT5	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	0.00	0.00	3.22

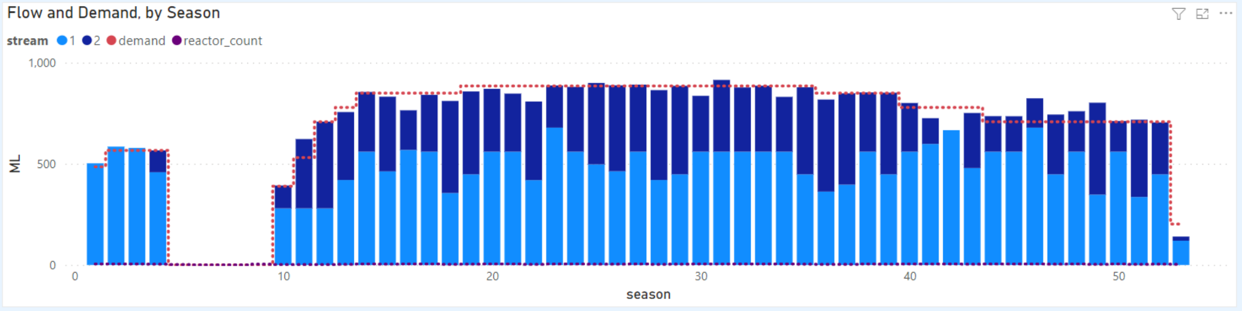
Visualisation – Power BI

Model Solution:

Demand and Bore (Well) Abstraction Schedule



Treatment Stream Proportion Schedule



Think climate change.
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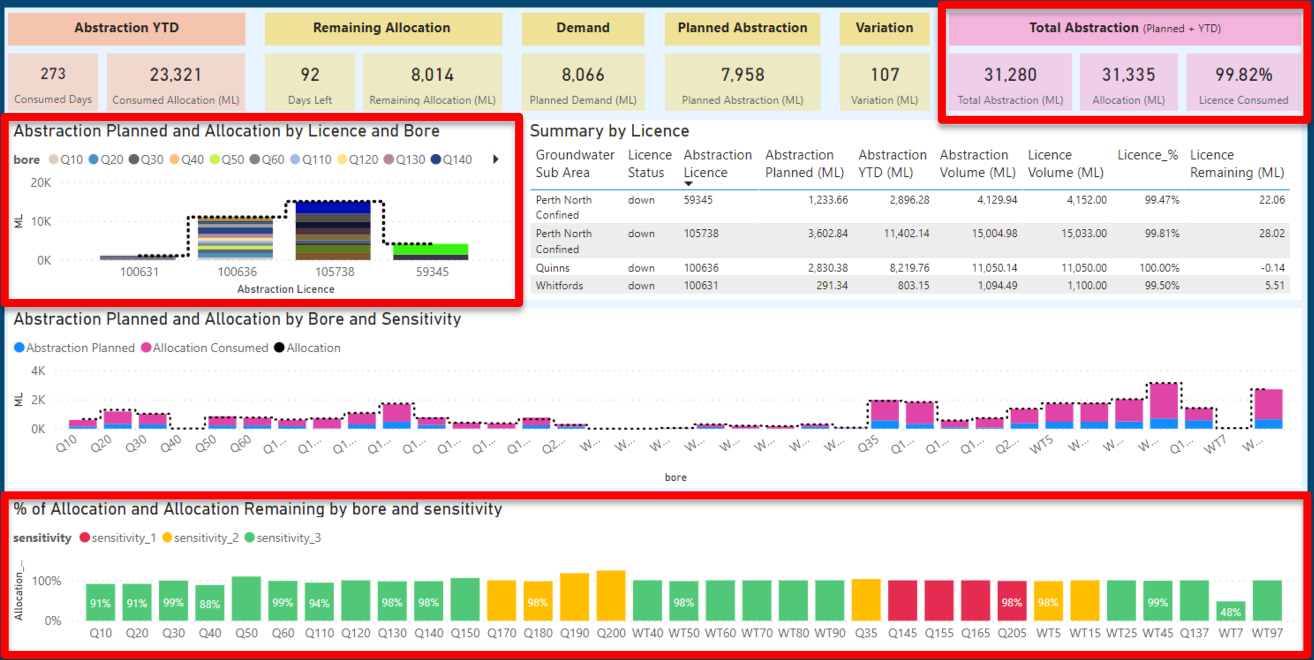
WATER CORPORATION

Visualisation – Power BI

Model Solution:

Scheduled Licence Allocation Summary

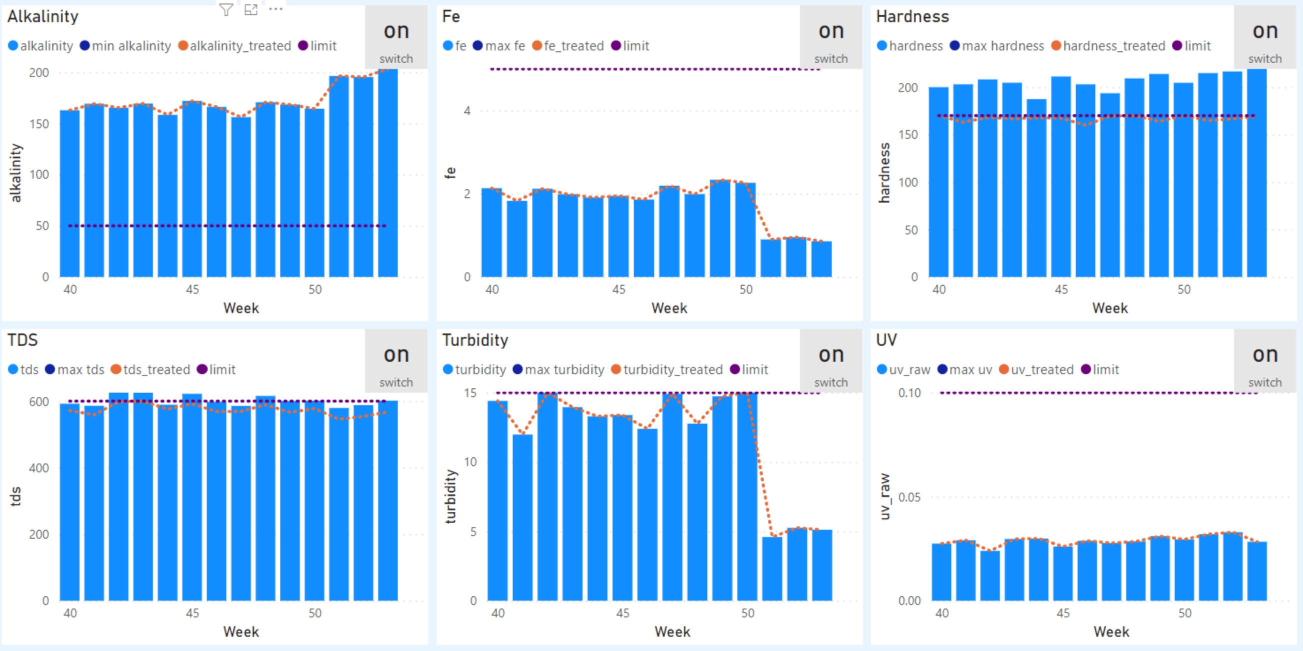


Groundwater Sub Area	Licence Status	Abstraction Licence	Abstraction Planned (ML)	Abstraction YTD (ML)	Abstraction Volume (ML)	Licence Volume (ML)	Licence_%	Licence Remaining (ML)
Perth North Confined	down	59345	1,233.66	2,896.28	4,129.94	4,152.00	99.47%	22.06
Perth North Confined	down	105738	3,602.84	11,402.14	15,004.98	15,033.00	99.81%	28.02
Quinns	down	100636	2,830.38	8,219.76	11,050.14	11,050.00	100.00%	-0.14
Whitfords	down	100631	291.34	803.15	1,094.49	1,100.00	99.50%	5.51

Visualisation – Power BI

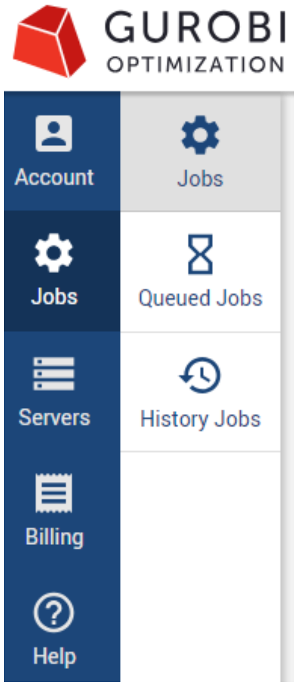
Model Solution:

Water Quality Profiles



Gurobi Solver

- Originally we used a free solver
  - Unable to solve our annual model for more than 3 months of an annual plan!
- Gurobi was purchased in 2020 able to solve the model most of the time.
- Performance tuning and upgraded versions of the solver can now get the models to solve in the order of minutes – some of our most complex now to the order of seconds.
- We now are using Gurobi for other projects and provide a user interface to run the optimisation model.



Model Formulation Issues

Originally our model had a weak, unnecessarily large formulation. This simplified some constraints but came with big performance issues.

We re-modelled, simplifying our decision variables. We introduced new auxiliary variables to help with complex logic and the result was a big performance improvement.

Don't be scared to try different model formulations

Model Performance Tuning

Feasible

- Focussed on **fast**, feasible solution
- Yields **low quality** results

Optimal

- Focussed on getting a **good** solution
- Can be slower, need for **tuning**

Model Performance Tuning

Initial tuning from learning materials and experience only got us so far

Used literature and documentation to inform how to improve

The **Parameter Tuning Tool** from Gurobi was a big help

- Identified important parameters
- Created parameter presets for individual models

Troubleshooting Infeasibility

User input issues

Easy to understand infeasibility

Pre-processing check, developed with user.

Necessary as the number of inputs is extensive.

IIS – Irreducible Inconsistent Subsystem

Developer analysis to determine the "offending constraints"

Often will identify the subset to analyse further.

Feasibility Relaxation

In built Gurobi tools to determine specific violated constraints.

Constraint Hunting (old way)

Guess and check, turn constraints on / off in the model.

As model has developed, infeasibility troubleshooting is less of an issue – we understand the model sensitivity more closely, and have more advanced tools to help diagnose.

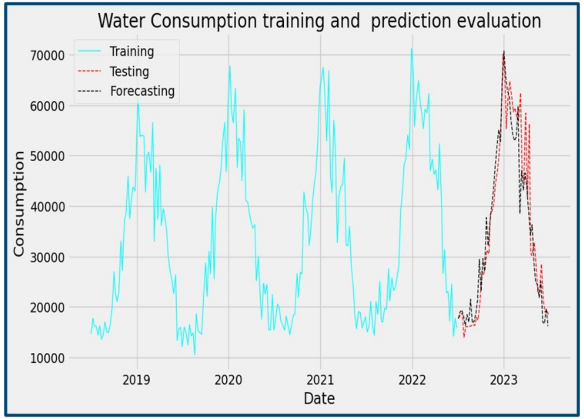
Further Work and Models

Related models

- Planning scenarios for asset sensitivity
- Groundwater bore (well) combination and timing for intra-day operation
- Water transfer and production model
- Hydraulic consideration into bore (well) activation selection
- Improve production demand forecasting

Other projects

- Energy contract optimisation



Outcomes and Benefits

Innovative Solution

Improved Automation

Optimised Solutions

Flexible & Dynamic

Licence Compliance

Chemical Cost Savings

Questions?

There are many people to thank who have been involved with our project since 2019, with special mention to:

- Amanda de Azevedo Marques
- Amanda McGillivray
- Christina Burt
- Kaveh Azizian
- Mike Canci