How Vodafone Uses Optimization to Tackle the Telecommunication Industry's Toughest Problems

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GUROBI OPTIMIZATION

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Today's Speaker

Rolf Bardeli, PhD

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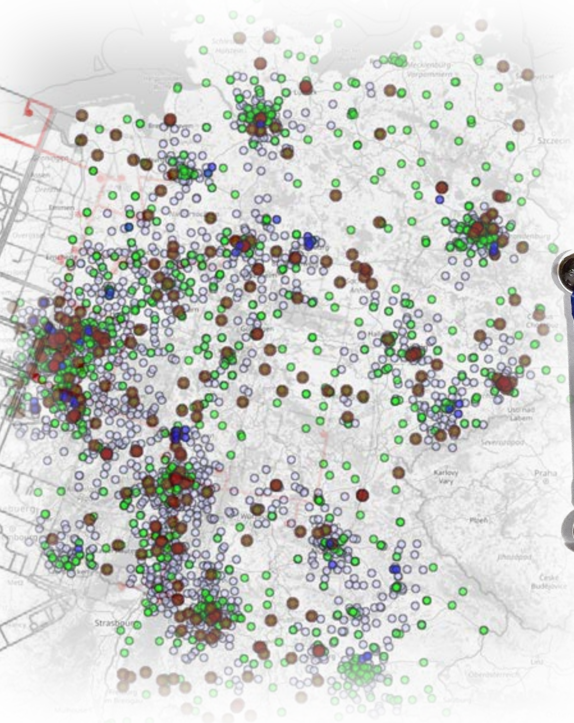
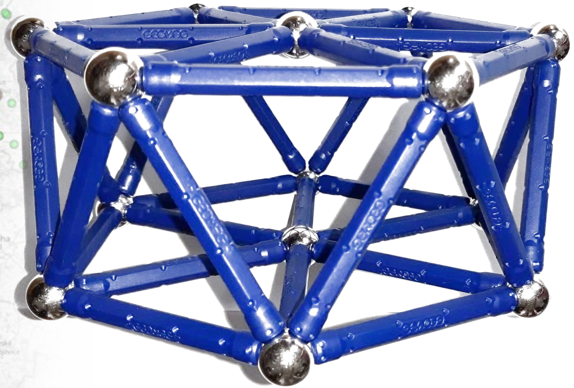
GraphILP API



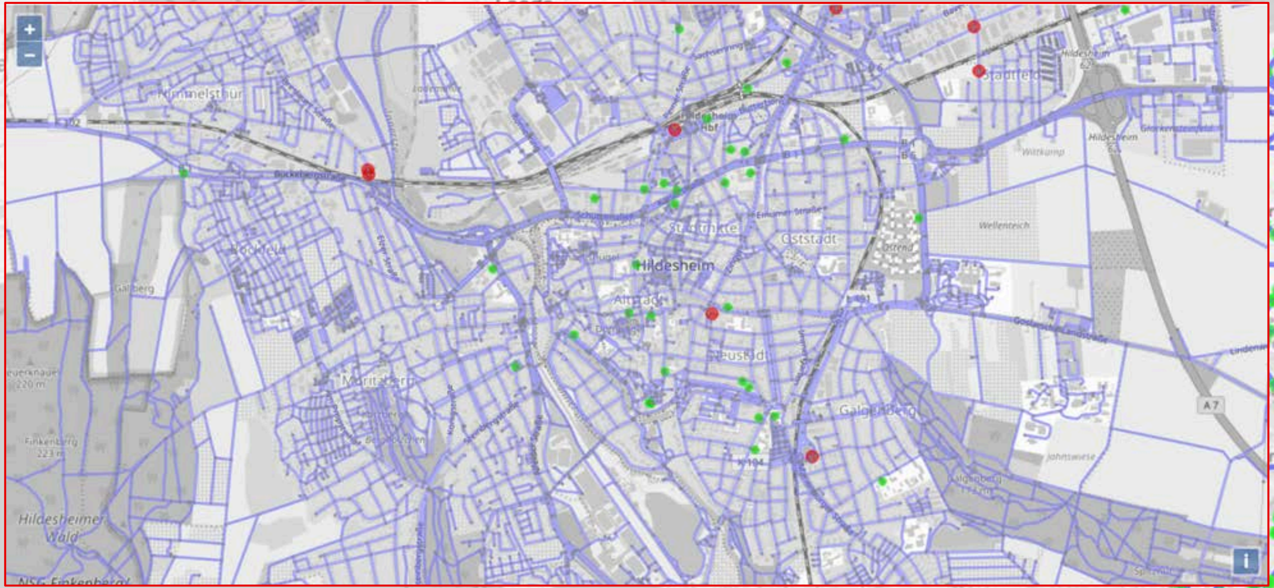


Optimal fibre networks

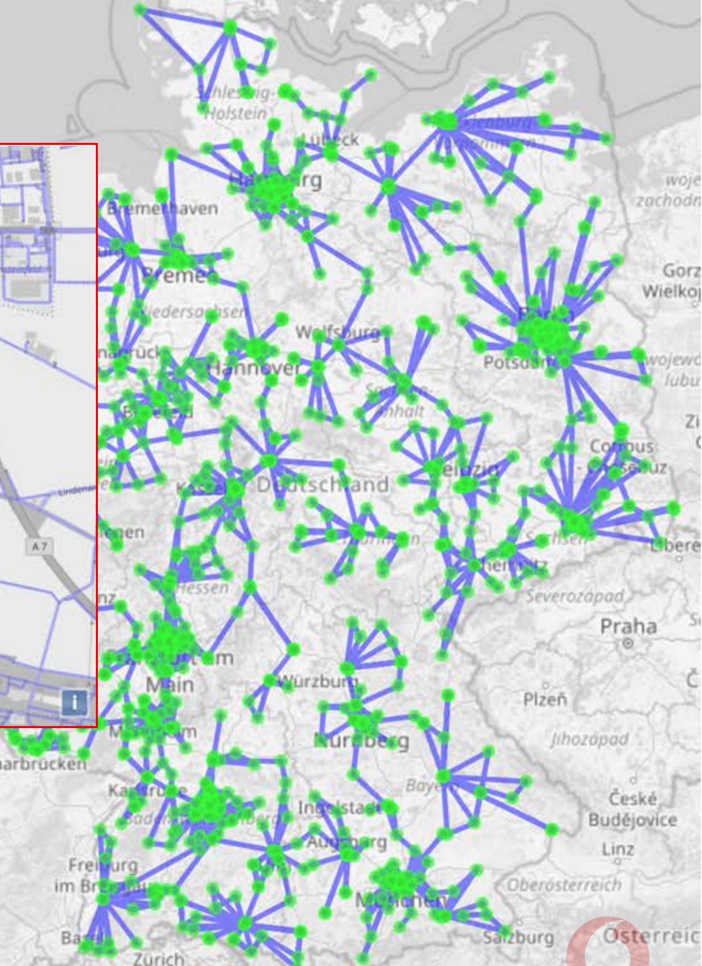
Optimal shop footprints

API for ILPs from graphs

Where the graphs are lurking



GRAPH UNDERLYING FIBRE OPTIMISATION



GRAPH UNDERLYING SHOP FOOTPRINT OPTIMISATION

From a graph $G = (V, E)$ to an integer linear program

Node variables $\boldsymbol{x_u}$

Edge variables $\boldsymbol{x_{uv}}$

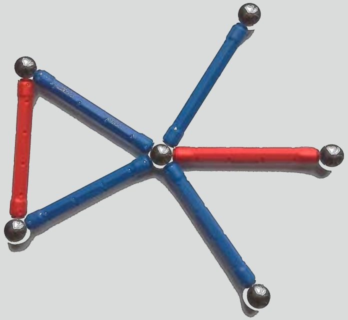
Labels $\boldsymbol{\ell_v}$

Example 1: **Max. weight matching**

$$\max \sum_{\{u,v\} \in E} w_{uv} \cdot x_{uv}$$

s.t.

$$\forall u \in V: \sum_{\{u,v\} \in E} x_{uv} \leq 1$$

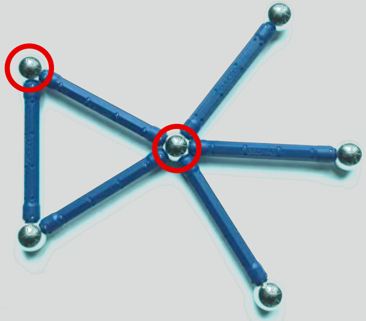


Example 2: **Min. vertex cover**

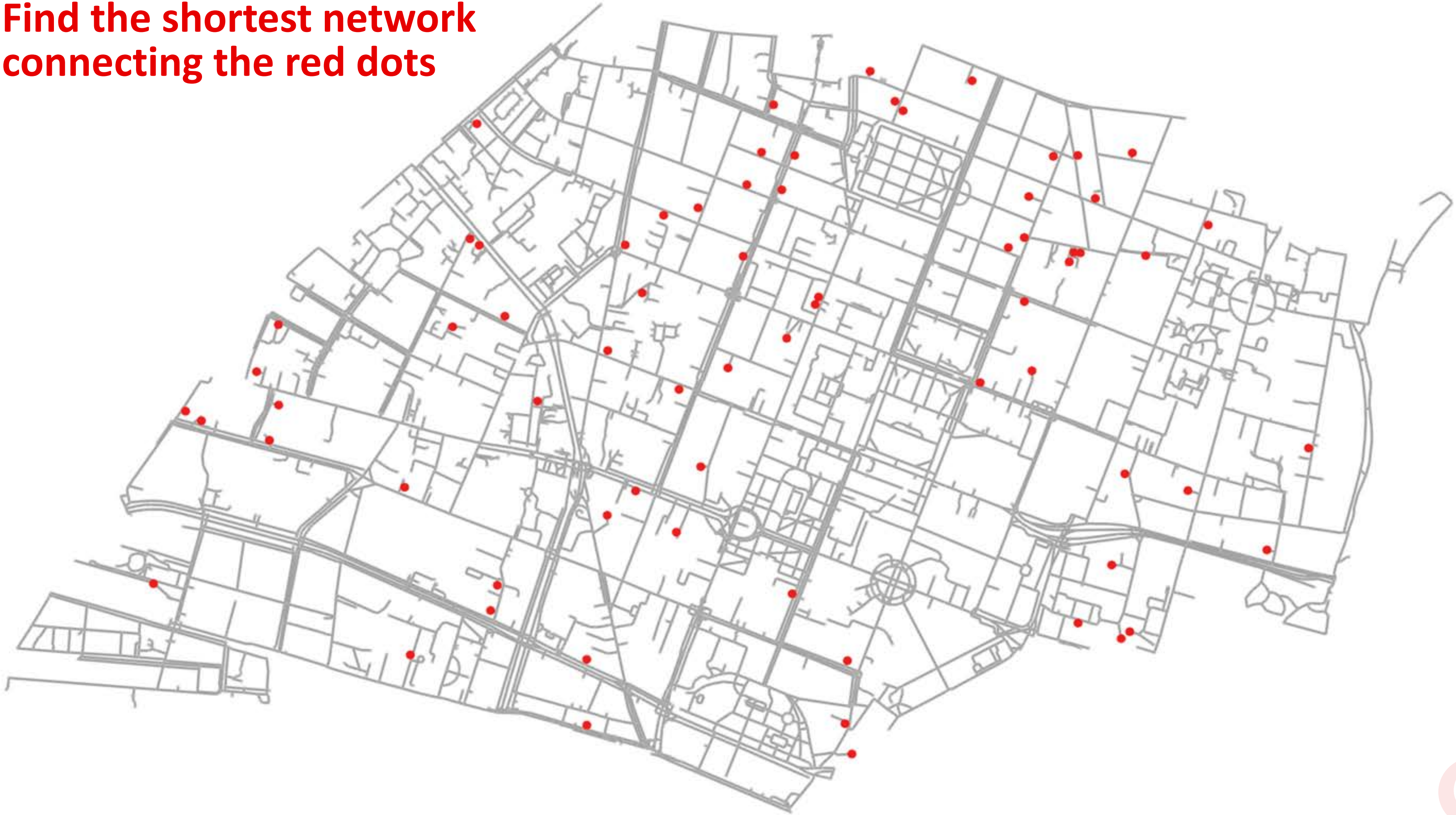
$$\min \sum_{u \in V} x_u$$

s.t.

$$\forall \{u, v\} \in E: \mathrm{x}_u + x_v \geq 1$$



Find the shortest network connecting the red dots



Steiner tree problem

Let $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$ be a weighted undirected graph, $|\boldsymbol{V}| =: \boldsymbol{n}$, and $\boldsymbol{T} \subseteq \boldsymbol{V}$ a set of **terminals** (red dots).

For $S \subset V$ define the **cut** $\boldsymbol{\delta(S)} \coloneqq \{(u,v) \in E : u \in S \wedge v \in V \setminus S\}$.

Exponential-size constraint system

$$\min \sum_{\{u,v\} \in E} w_{uv} \cdot x_{uv}$$

s.t.

$$\forall t \in T: x_t = 1$$

$$\forall \{u,v\} \in E:\ 2x_{uv} - x_u - x_v \leq 0$$

$$\forall v \in V: x_v - \sum_{\{u,v\} \in E} x_{uv} \leq 0$$

$$\forall S \subset V: \forall w \in S: \sum_{\{u,v\} \in \delta(S)} x_{uv} \geq x_w$$

Linear-size constraint system

$$\min \sum_{\{u,v\} \in E} w_{uv} \cdot x_{uv}$$

s.t.

$$\forall t \in T: x_t = 1$$

$$\forall \{u,v\} \in E:\ 2x_{uv} - x_u - x_v \leq 0$$

$$\forall v \in V: x_v - \sum_{\{u,v\} \in E} x_{uv} \leq 0$$

$$x_{uv} + x_{vu} \leq 1$$

$$\sum_{v \in V} x_v - \sum_{\{u,v\} \in E} x_{uv} = 1$$

$$\forall \{u,v\} \in E:\ n x_{uv} + \ell_v - \ell_u \geq 1 - n(1 - x_{vu})$$

$$\forall \{u,v\} \in E: n x_{vu} + \ell_u - \ell_v \geq 1 - n(1 - x_{uv})$$

Trade-off

Use linear-size constraint system

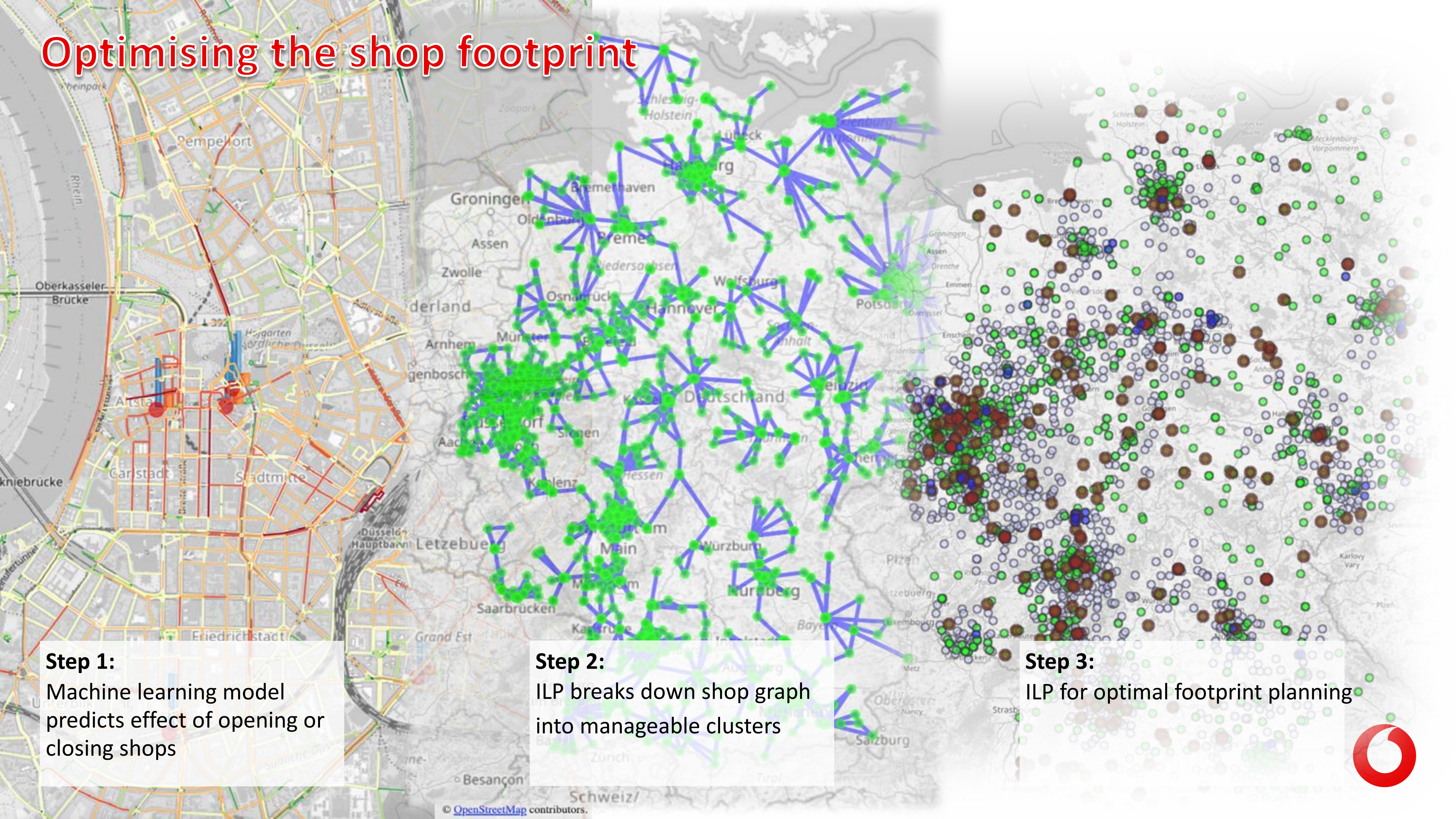
+

Add necessary conditions for tree:

- Forbid cycles from cycle basis,
- Add colouring constraints.

? Can we run these in parallel and exchange lower and upper bound info?

Optimising the shop footprint

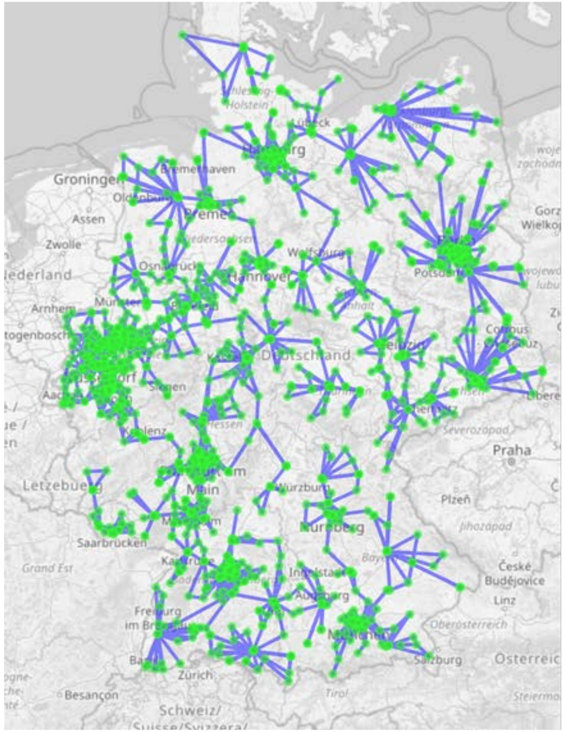


Step 1:
Machine learning model predicts effect of opening or closing shops

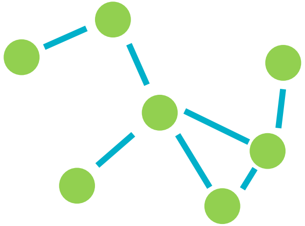
Step 2:
ILP breaks down shop graph into manageable clusters

Step 3:
ILP for optimal footprint planning

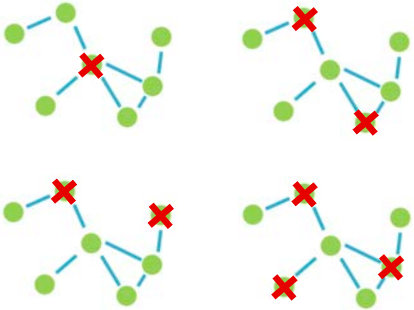
Machine learning + ILP = optimal shop footprint



Clustering ILP



Closing scenarios



Optimisation scenario (example)

$$\max \sum_{cluster} value(cluster) \times cluster\_var$$

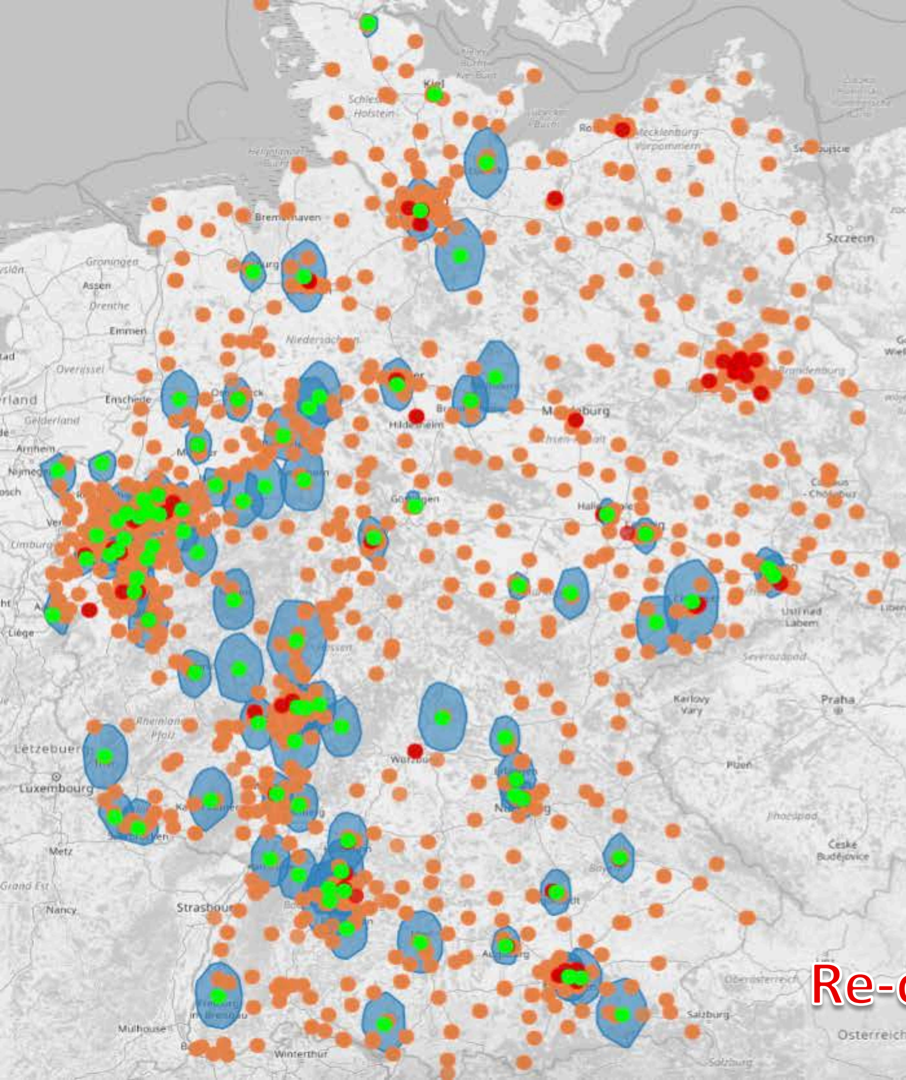
s.t.

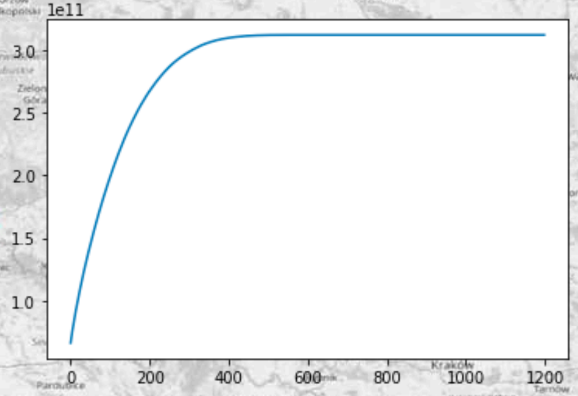
$$\sum_{shop} shop\_var \leq (1-\epsilon)\#shops$$

Per shop → shop_attribute x shop_var

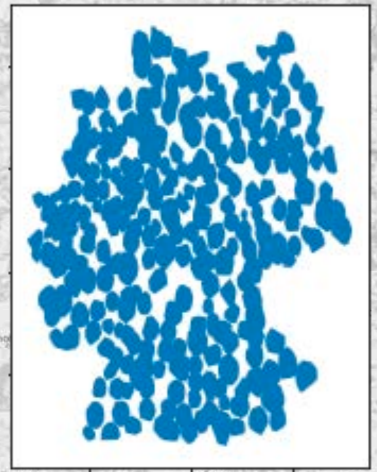
Per cluster → cluster_attribute x cluster_var
(from machine learning)

Re-opening shops after Covid lockdown





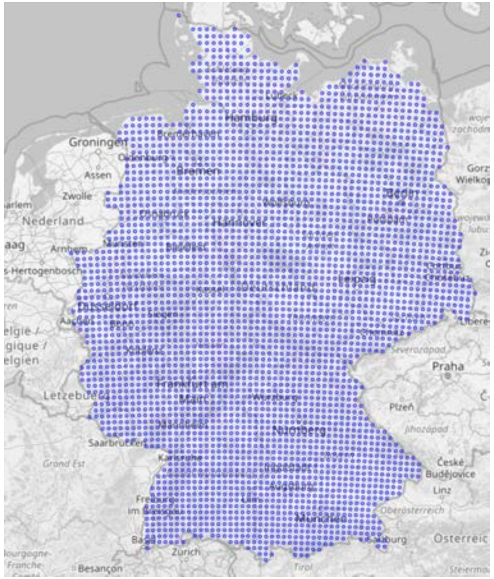
Open shops vs. increase of coverage



Example coverage for 200 shops

Definition: Max k-Coverage Problem

Instances: $V = \{1, 2, ..., n\}$ and $\mathcal{S} \subseteq 2^V$, $k \in N$
Solutions: $S \subseteq \mathcal{S}$ with $|S| \leq k$
Task: Maximize $|\cup_{F_i \in S} F_i|$



ILP- Formulation for the Max k-Coverage Problem

$$\max \sum_{j \in V} x_j,$$

s. t.

$$\sum_{i: j \in F_i} y_i \geq x_j, \quad \text{for each } j \in V$$

$$\sum_{i: F_i \in \mathcal{S}} y_i \leq k,$$

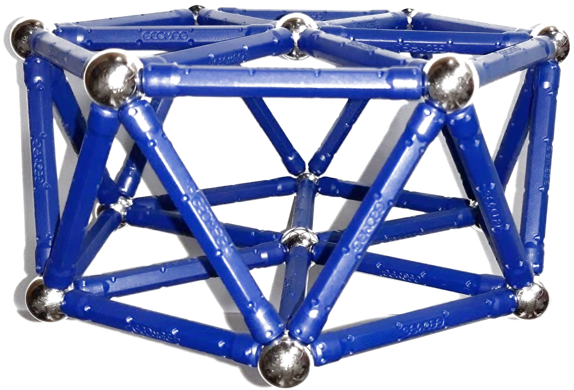
$$y_i \in \{0, 1\} \text{ for each } F_i \in \mathcal{S}$$

$$x_j \in \{0, 1\} \text{ for each } j \in V$$

Some facts about Max k Coverage

1. Max-k-Coverage Problem is NP-hard
2. There is a natural ILP formulation for the problem.
3. Approximation below a factor of 1.582 is NP-hard.
4. Approximation ratio attained by the greedy algorithm is best possible unless $P = NP$.
5. The integrality gap of the natural ILP is 1.582.

GraphILP API



GraphILP API

Clusters and communities

Facility Location Problem

Factors

Spanning tree / forest

Partitioning

Vertex colouring

Cuts and flows

Max flow
Min k-flow
Min/max cut
Min uncut
Min/max bisection

Matching

Max weight matching
Bipartite matching (LP)

Steiner

Steiner tree
Prize collecting

Covering

Set Cover
Vertex Cover

Packing

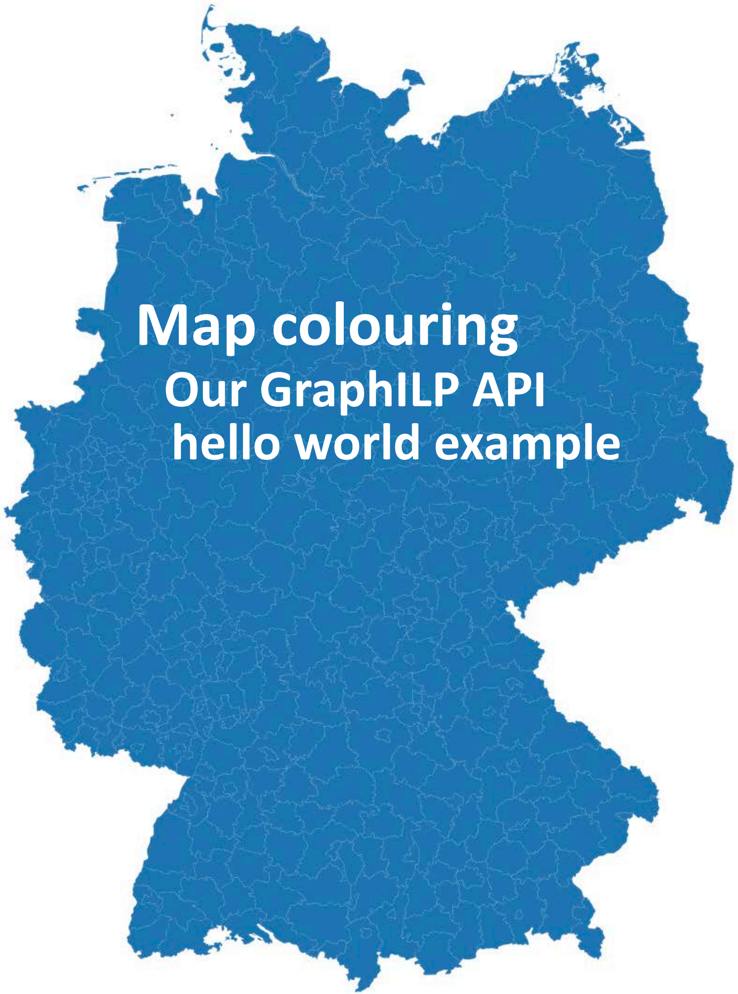
Max-k-coverage
Set packing
Max clique
Max independent set

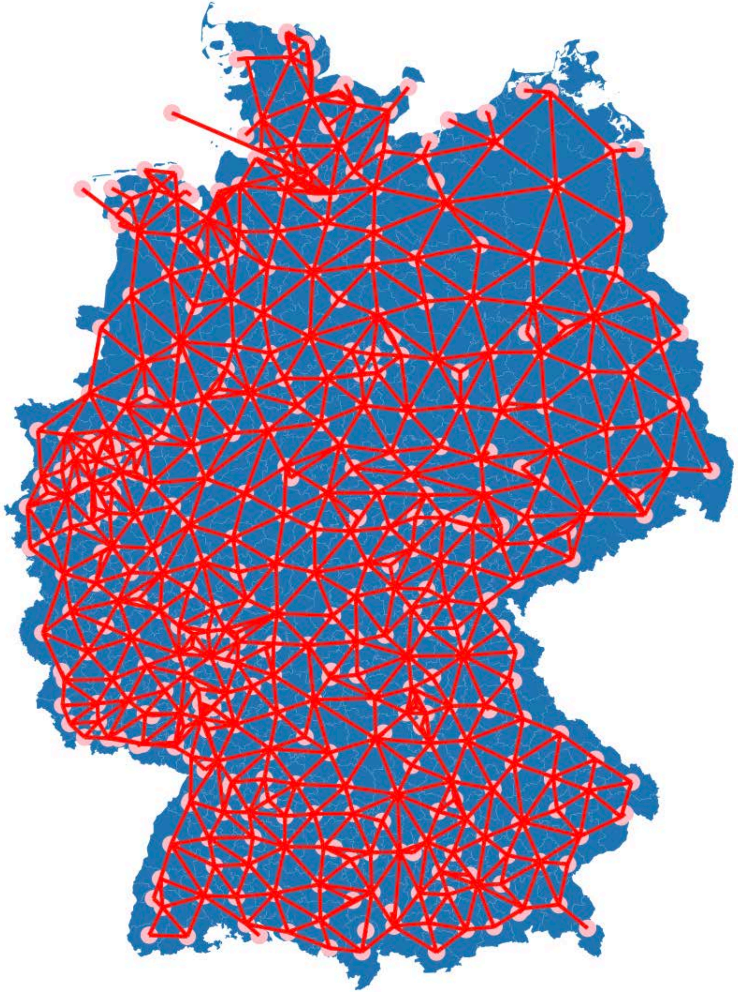
TSP

Map colouring

Our GraphILP API

hello world example





Min Vertex Colouring

GraphILP API

Importing GraphILP API

```
# We will need a helper to import the Networkx graph
from graphilp.imports import networkx as imp_nx


# Here is the module doing the map colouring
from graphilp.partitioning import min_vertex_coloring
```

Solving the problem

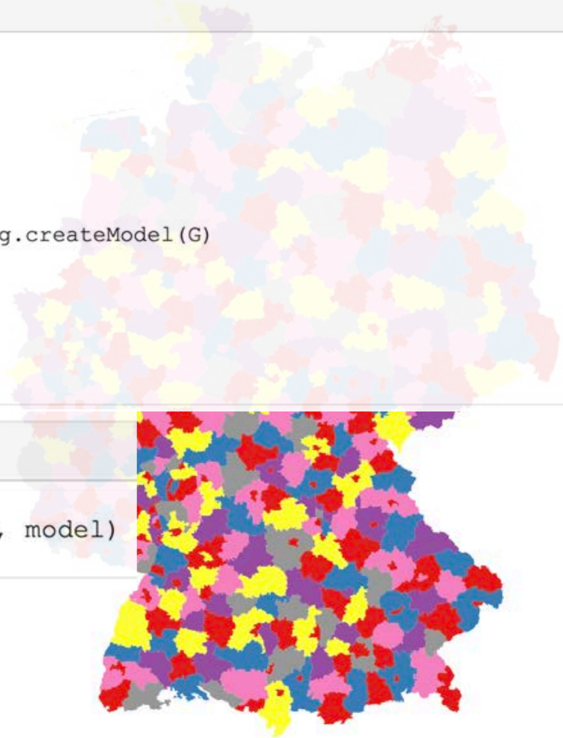
```
# import the graph
G = imp_nx.read(mygraph)


# set up an ILP model
model = min_vertex_coloring.createModel(G)


# compute the result
model.optimize()
```

Extracting the solution

```
color_assignment, node_to_col = min_vertex_coloring.extractSolution(G, model)
```



GraphILP API

Max Clique

Import the max clique module

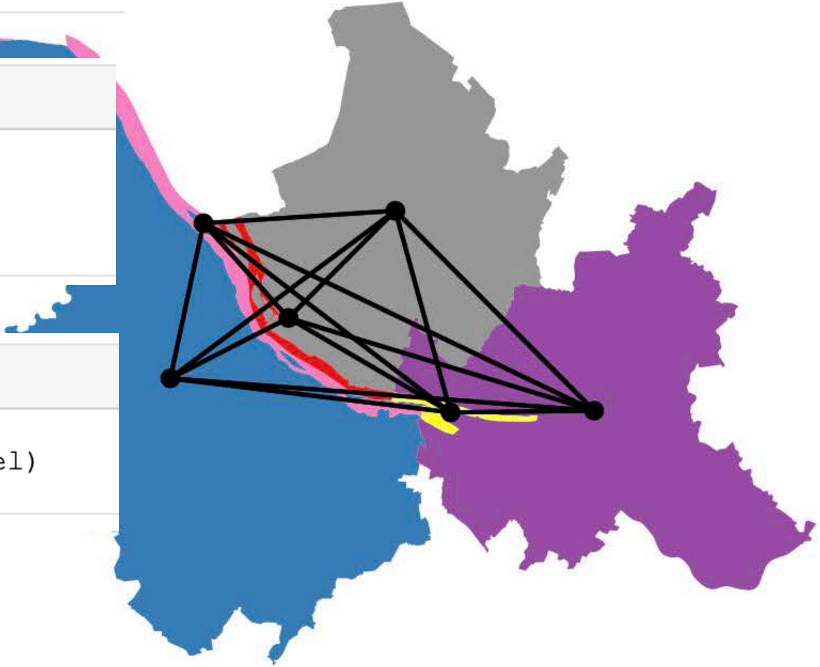
```
from graphilp.packing import max_clique
```

Solving the max clique problem

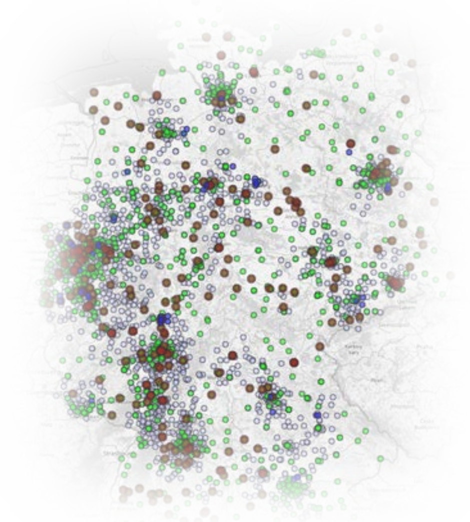
```
model = max_clique.createModel(G)
model.optimize()
```

Extracting the max clique

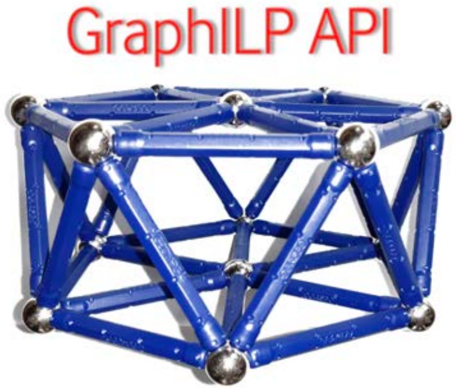
```
clique = maxclique.extractSolution(G, model)
```



Optimal fibre networks



Optimal shop footprints



API for ILPs from graphs

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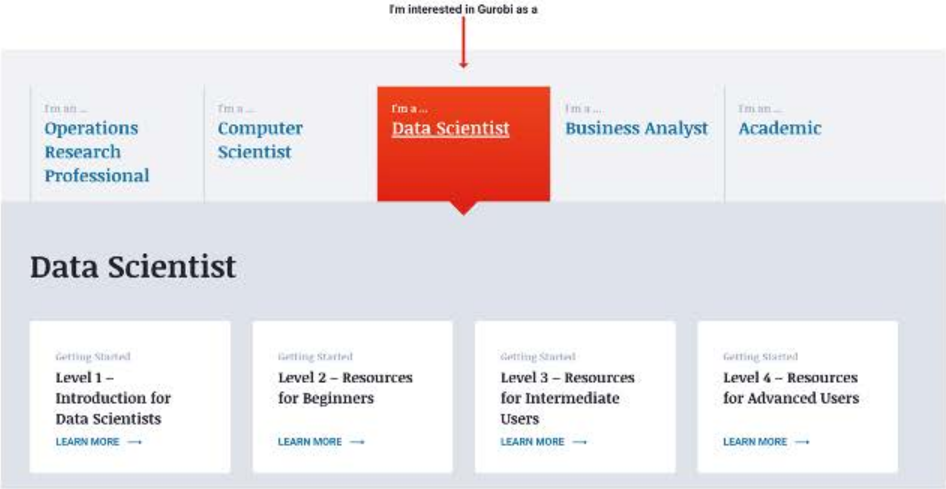

Modeling Examples

- **Jupyter Notebook Modeling Examples**
  - Cell Tower Coverage
  - Customer Assignment
  - Facility Location
  - Feature Selection for Forecasting (L0-Regression)
  - Offshore Wind Farming
  - Standard Pooling
  - Traveling Salesman
  - And many more!
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  - Visit our virtual booth to chat with experts and for a fun modeling examples challenge.

Thank You

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