OPTIMIZING YOUR FINANCIAL FUTURE

POWERED BY GUROBI OPTIMIZATION

GUROBI
OPTIMIZATION

mygoals

INTRODUCTION

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- Founder, MyGoals

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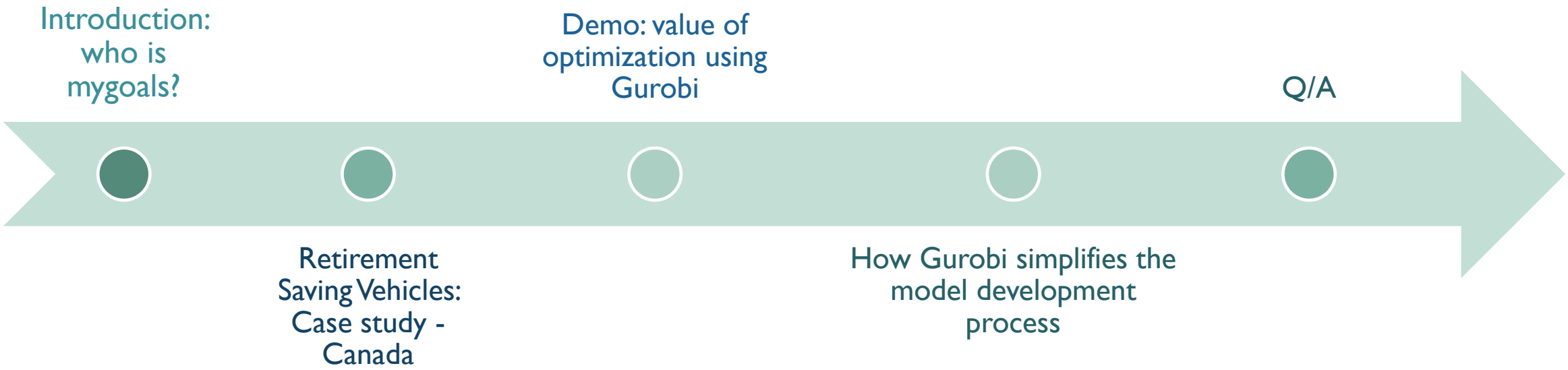
CONTENT


Introduction: who is mygoals?
Retirement Saving Vehicles: Case study - Canada
Demo: value of optimization using Gurobi
How Gurobi simplifies the model development process
Q/A

mygoals

Mygoals is a Toronto, Canada based company that has built an entirely unique financial planning platform based on mixed integer optimization.

We combine all sources of savings and income and prioritize how investors should accumulate and decumulate their wealth.

We designed the software to help as many Canadians as possible across all levels of income and assets.

We track and prioritize all your financial goals and help you achieve them all!

We are now looking to grow our team.

What we discovered has international implications and potential

Gurobi's technology was an essential building block for us.

THINK OF US AT THE INTERSECTION OF …

BUILT FOR BOTH SPEED AND ACCURACY

SPEED

Account prioritization are outputs rather than inputs

ACCURACY

Our investor benchmarked mygoals versus other financial planning tools. Mygoals outperformed.

The more complex the plan, the more value we added

- CCPC
- Income splitting
- Complex cashflow analysis

THE VALUE OF OPTIMIZATION

mygoals adds on average between 2 - 10% + PER ANNUM of post retirement, after-tax income for **"most"** naïve financial assumptions.

Mygoals typically adds 5% annually for simple plans and 20%+ annually for complex plans. Benchmarking performed by our angel investor independently.

General rules of thumb never apply.

RETIREMENT SAVING VEHICLES

TFSA

Tax Free Saving Account, similar to **Roth IRA** in US

NON REGISTERED SAVINGS

After tax savings with dividend and capital gains implications

RRSP

Registered Retirement Saving Plan, similar to **401k**

CPP

Canada Pension Plan, similar to **Social Security** like pay as you go plan

OAS

Old Age Security, secondary pension based solely on Canadian residency, taxable and clawed back

GIS

Guaranteed Income Supplement, retirement welfare, low income supplement

RETIREMENT SAVING VEHICLES

LIRA

Locked-in Retirement Account, similar to **401k** but has min and max withdrawal

CCPC

Canadian Controlled Private Corporation, holding company with 4 methods of payments to owners

DEFINED BENEFIT

Defined Benefit Pension Plan

GROUP RRSP

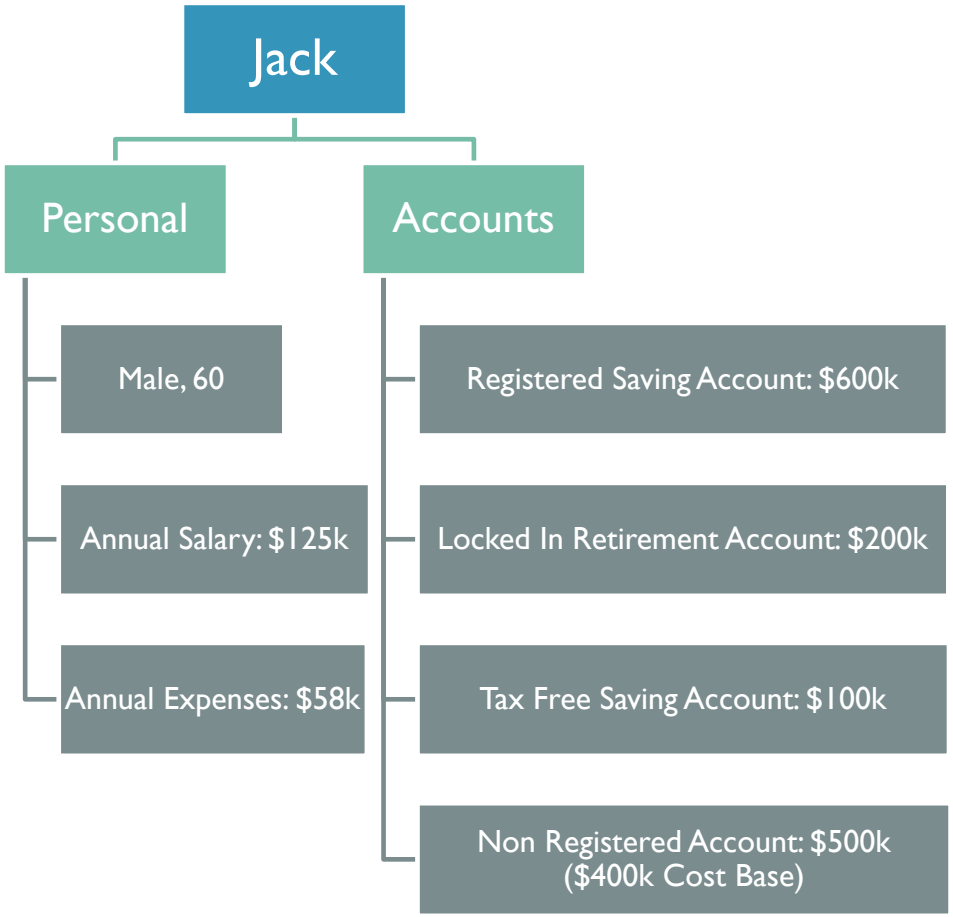
Similar to a defined contribution plan with matching

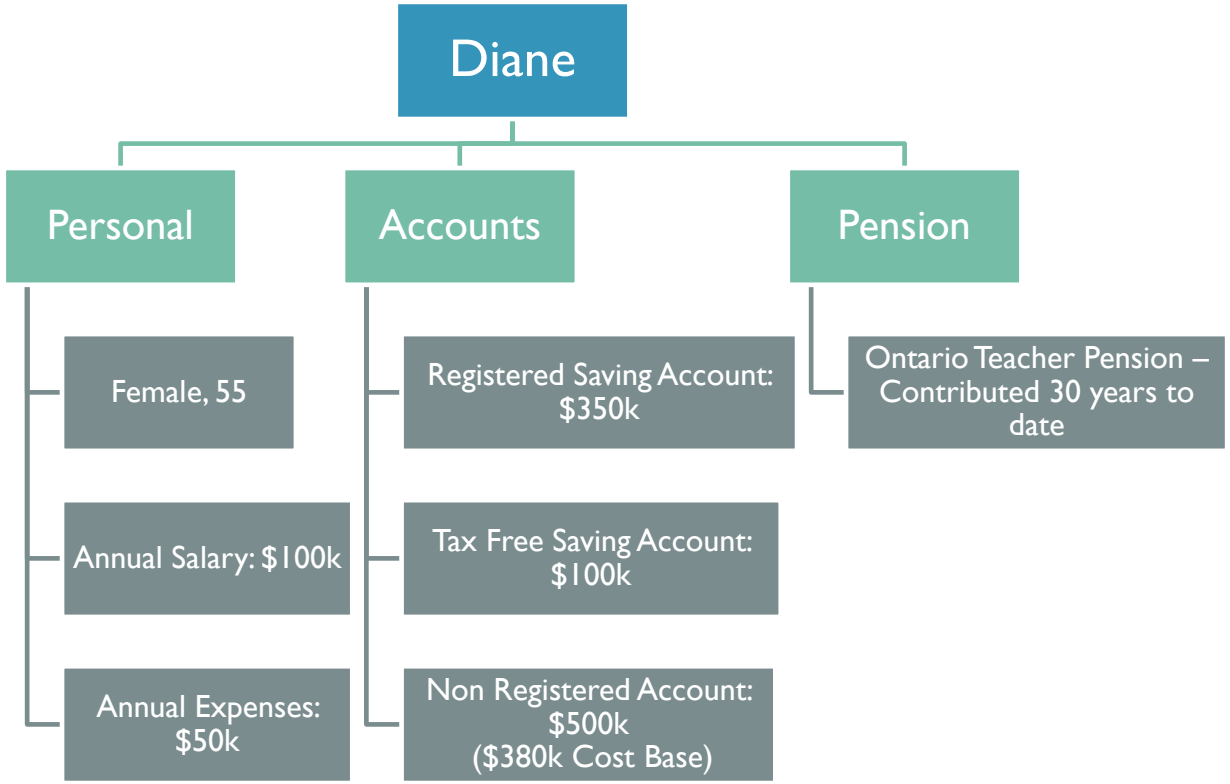
Efficient income splitting with spouse – **only available to retirees in Canada but can be split by all couples in the US**

We optimize both accumulations while clients are working, and decumulations to fund retirement

Canada and US have "progressive tax regimes". As you earn more you pay a larger % in tax

DEMO – SCENARIO EXAMPLE – GUROBI FAMILY





DEMO – SCENARIO EXAMPLE – GUROBI FAMILY

Government Benefits:

- CPP: Not yet receiving, expecting full benefit
- OAS: Not yet receiving, expecting full benefit

Retirement Plan:

- Both want to retire at 65
- Plan a life expectancy of 95

Desired Outcome:

- Maximize Retirement After Tax Income

Portfolio Profile:

- Mean Returns: 4.53%
- Volatility: 8.51%

DEMO – VALUE OF OPTIMIZATION

By solving the financial plan model with Gurobi for:

- The optimal cashflow withdrawal strategy
- The optimal age to elect and receiving government entitlements

Pre-Optimized Income

$206,000

By Optimizing with

We found you an additional:

$12,849 (6.24%) / year, risk free

Over the duration of your retirement plan, optimization could save you a total of **$385,470**

See how we did it

Optimized Income

$218,849

HOW GUROBI SIMPLIFIES MODEL DEVELOPMENT PROCESS

PIECEWISE LINEAR CONSTRAINT

MIN/MAX CONSTRAINT

SOLVER CALLBACK

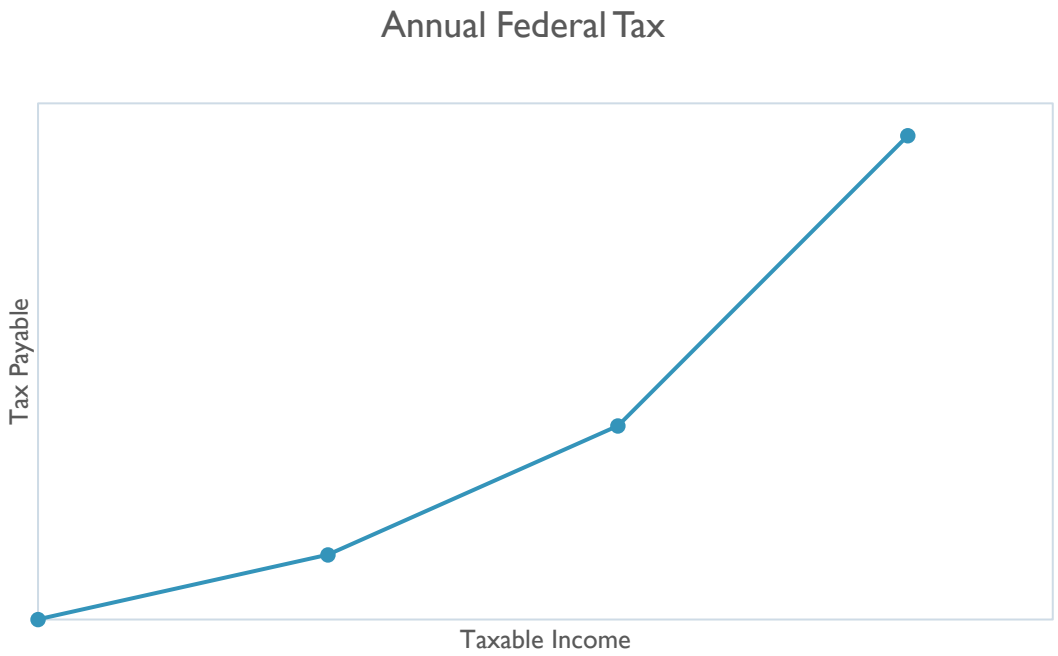
IRREDUCIBLE INCONSISTENT SUBSYSTEM (IIS)

HOW GUROBI SIMPLIFIES MODEL DEVELOPMENT PROCESS

PIECEWISE LINEAR CONSTRAINT – WITHOUT GUROBI

We consistently use piecewise linear constraints in our model.

- One such example is to calculate the annual tax payable given a taxable income.



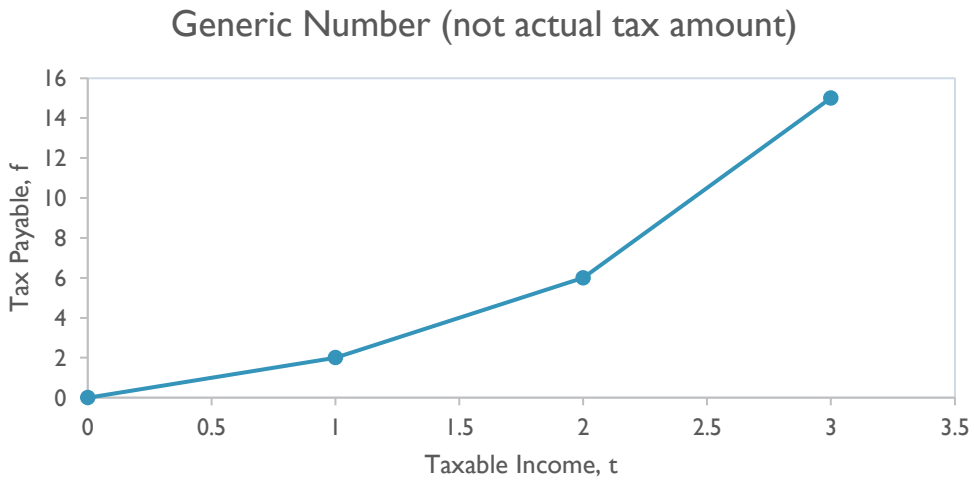
HOW GUROBI SIMPLIFIES MODEL DEVELOPMENT PROCESS

PIECEWISE LINEAR CONSTRAINT – WITHOUT GUROBI

For example:

- Let t_i be the taxable income, and f_i be the tax payable.
- Let $b_1, b_2, \ldots, b_{n-1}$ be binary variables such that $b_i \in \{0, 1\}$
- Let $s_1, s_2, \ldots, s_{n-1}$ be segment variables such that $s_i \in R$

We have $t = [0, 1, 2, 3]$ and $f = [0, 2, 6, 15]$



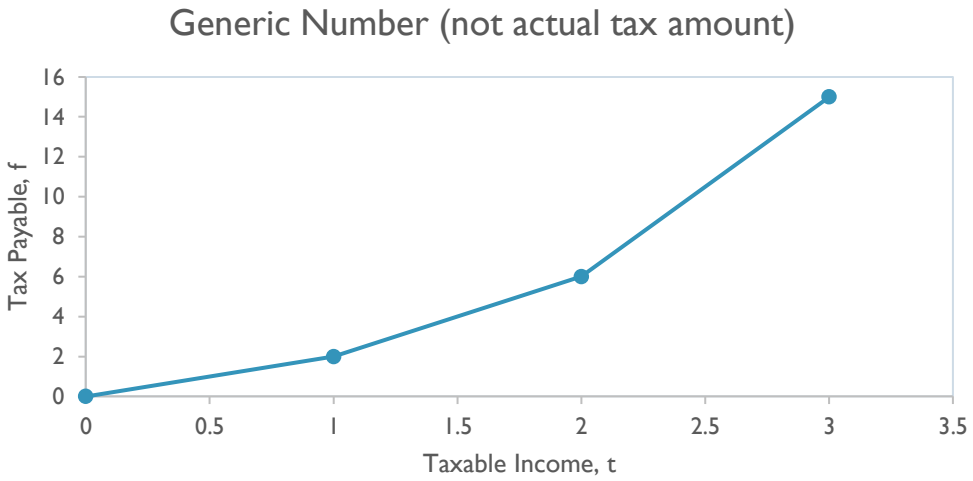
HOW GUROBI SIMPLIFIES MODEL DEVELOPMENT PROCESS

PIECEWISE LINEAR CONSTRAINT – WITHOUT GUROBI

This piecewise linear function can be modelled with the following constraints:

- $\boldsymbol{b_1 + b_2 + b_3 = 1}$
- $\boldsymbol{0 \leq s_i \leq b_i \quad for\ i = 1, 2, 3}$
- $t = t_1 b_1 + (t_2 - t_1)s_1 + t_2 b_2 + (t_3 - t_2)s_2 + t_3 b_3 + (t_4 - t_3)s_3$
- $t = 0 \cdot b_1 + (1 - 0)s_1 + 1 b_2 + (2 - 1)s_2 + 2 b_3 + (3 - 2)s_3$
- $\boldsymbol{t = s_1 + b_2 + s_2 + 2b_3 + s_3}$
- $f = f_1 b_1 + (f_2 - f_1)s_1 + f_2 b_2 + (f_3 - f_2)s_2 + f_3 b_3 + (f_4 - f_3)s_3$
- $f = 0 \cdot b_1 + (2 - 0)s_1 + 2 b_2 + (6 - 2)s_2 + 6 b_3 + (15 - 6)s_3$
- $\boldsymbol{f = 2s_1 + 2b_2 + 4s_2 + 6b_3 + 9s_3}$

$t_i = [0, 1, 2, 3]$ and $f_i = [0, 2, 6, 15]$



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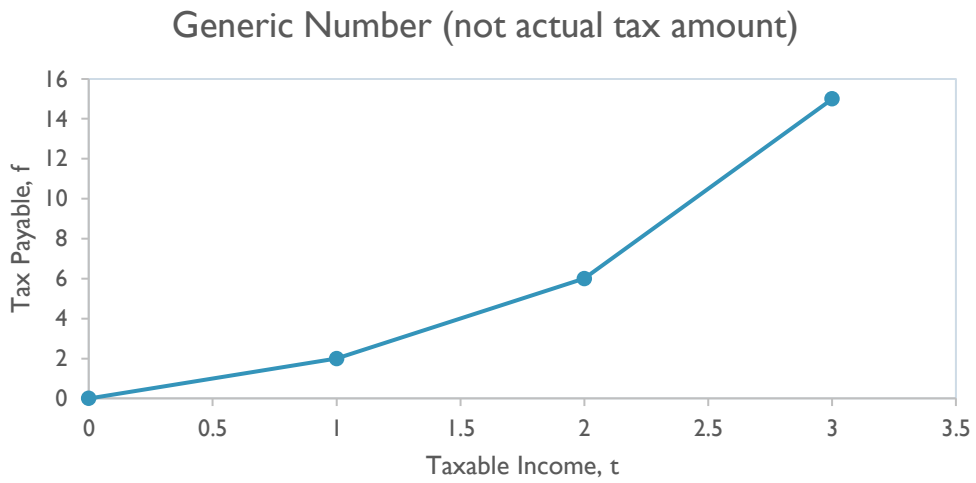
PIECEWISE LINEAR CONSTRAINT – WITHOUT GUROBI

This piecewise linear function can be modelled with the following constraints:

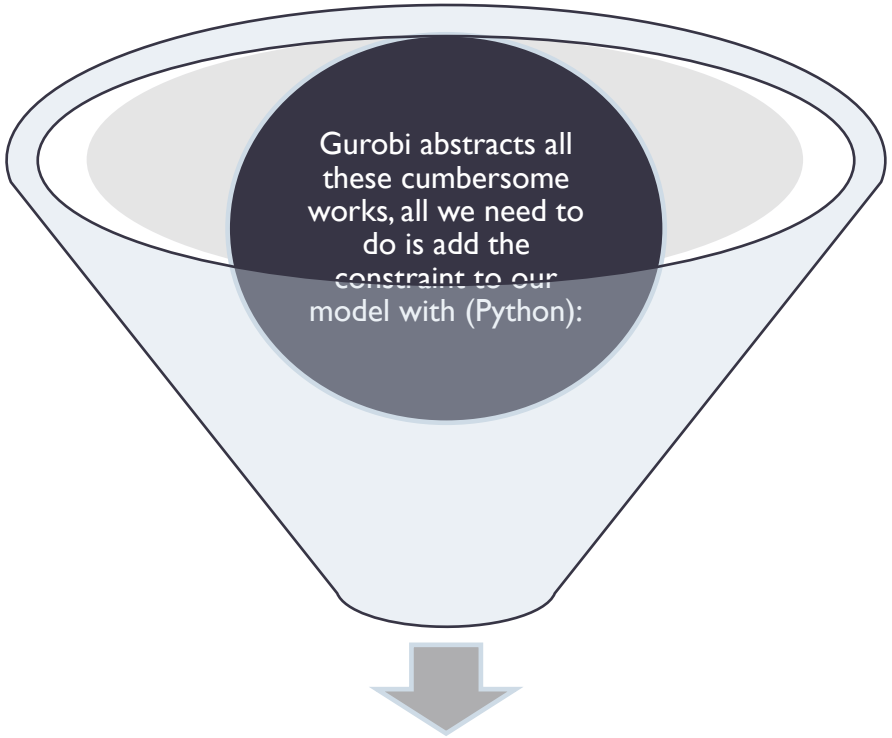
- $b_1 + b_2 + b_3 = 1$
- $0 \leq s_1 \leq b_1$
- $0 \leq s_2 \leq b_2$
- $0 \leq s_3 \leq b_3$
- $t = s_1 + b_2 + s_2 + 2b_3 + s_3$
- $f = 2s_1 + 2b_2 + 4s_2 + 6b_3 + 9s_3$

$t_i = [0, 1, 2, 3]$ and $f_i = [0, 2, 6, 15]$



HOW GUROBI SIMPLIFIES MODEL DEVELOPMENT PROCESS

PIECEWISE LINEAR CONSTRAINT – WITH GUROBI




```
model.addGenConstrPWL(xvar=t, yvar=f, xpts=[0, 1, 2, 3], ypts=[0, 2, 6, 15])
```

HOW GUROBI SIMPLIFIES MODEL DEVELOPMENT PROCESS

MIN/MAX CONSTRAINT – WITHOUT GUROBI

We also employ min/max constraint in our model – for example $z = \max(x, y)$, where x, y are decision variables.

Without Gurobi, we would have to create the following set of constraints to achieve this:

- $x - y \leq Mb$
- $y - x \leq M(1 - b)$
- $z \geq x$
- $z \geq y$
- $z \leq x + M(1 - b)$
- $z \leq y + Mb$

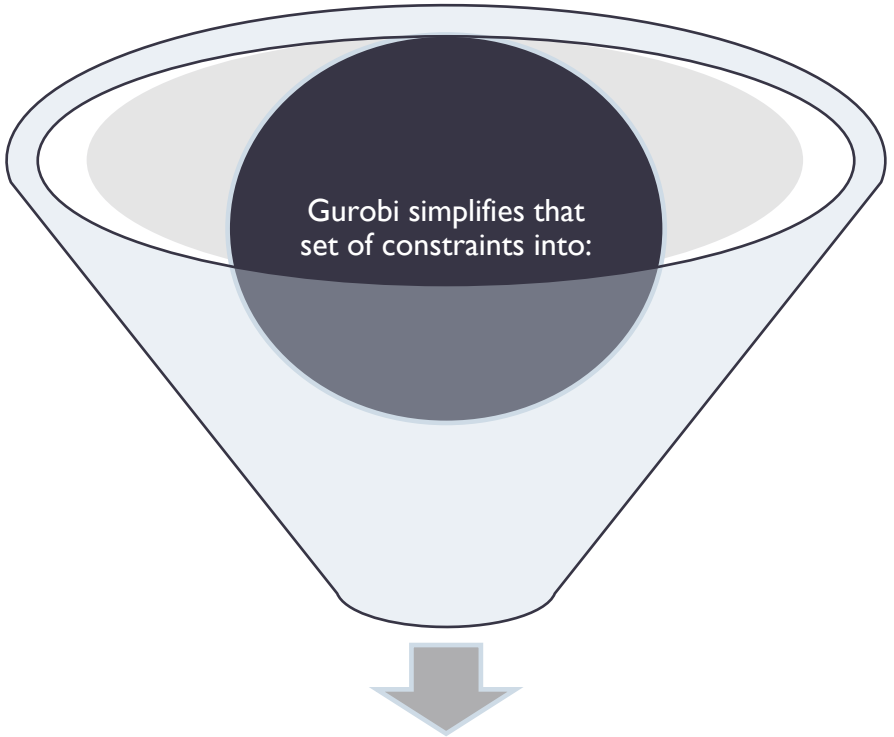
where b is a binary variable, and M is a constant such that $x, y <= M$ in a reasonable solution to the problem.

For example:

- $z = \max(5, 10)$
- $y > x, implies\ b = 0$

HOW GUROBI SIMPLIFIES MODEL DEVELOPMENT PROCESS

MIN/MAX CONSTRAINT – WITH GUROBI




```
model.addConstr(z == max_(x, y))
model.addConstr(z == min_(x, y))
```

HOW GUROBI SIMPLIFIES MODEL DEVELOPMENT PROCESS

SOLVER CALLBACK

Gurobi callbacks can be used both to monitor the progress of the optimization, obtain various information about the progress, and to modify the behavior of the Gurobi optimizer as well.

```
Gurobi Optimizer version 9.5.1 build v9.5.1rc2 (win64)
Thread count: 8 physical cores, 16 logical processors, using up to 16 threads
Optimize a model with 5219 rows, 6975 columns and 15375 nonzeros
Model fingerprint: 0x2b8465a4
Model has 1925 general constraints
Variable types: 6975 continuous, 0 integer (0 binary)
Coefficient statistics:
  Matrix range     [6e-04, 2e+00]
  Objective range  [1e+00, 1e+04]
  Bounds range     [2e+06, 3e+06]
  RHS range        [5e+01, 6e+05]
  PWLCon x range   [0e+00, 2e+05]
  PWLCon y range   [0e+00, 5e+04]
Presolve added 0 rows and 1591 columns
Presolve removed 422 rows and 0 columns
Presolve time: 0.09s
Presolved: 4797 rows, 8566 columns, 26618 nonzeros
Presolved model has 399 SOS constraint(s)
Variable types: 7877 continuous, 689 integer (689 binary)

Root relaxation: objective 6.744505e+06, 8898 iterations, 0.88 seconds (1.57 work units)
```

```
    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

    0     0 6744504.79    0  238          - 6744504.79      -     -    1s
    0     0 6744181.72    0  238          - 6744181.72      -     -    2s
    0     2 6744181.72    0  238          - 6744181.72      -     -    2s
 2552  2637 6657749.47  144  136          - 6729870.91      -  16.7    5s
 7542  5100 6673523.28   72  582          - 6729870.91      -  11.5   12s
 7545  5105 6729870.91   14  502          - 6729870.91      -   2.3   17s
 8862  6072 6687445.52   61  508          - 6729870.91      -   4.8   20s
11329  7537 6671491.54  176  324          - 6729870.91      -   6.3   27s
12401  8170 6675055.48   97  444          - 6729870.91      -   7.5   30s
13036  8487 6672731.55  114  413          - 6729870.91      -  10.9   35s
15942 10302 6669916.25  421  147          - 6729870.91      -  11.3   40s
19517 12491 6669965.15  104  431          - 6729870.91      -  10.5   45s
------------ Improved solution of 100.0% found at 48s ------------
*21313 11149             615    6665311.7722 6729870.91  0.97%  10.5   48s

Explored 21330 nodes (320229 simplex iterations) in 48.04 seconds (64.64 work units)
Thread count was 16 (of 16 available processors)

Solution count 1: 6.66531e+06

Optimal solution found (tolerance 5.00e-02)
Best objective 6.665311772207e+06, best bound 6.729870908421e+06, gap 0.9686%

User-callback calls 48807, time in user-callback 0.13 sec
*** Termination Status *** OPTIMAL
```

HOW GUROBI SIMPLIFIES MODEL DEVELOPMENT PROCESS

IRREDUCIBLE INCONSISTENT SUBSYSTEM (IIS)

An IIS is a subset of constraints and variable bounds with the properties that, it is still infeasible and if a single constraint or bound is removed, then the subsystem becomes feasible.

While developing the model, we sometime run into situation where the model is infeasible.

Since our model is large, it is difficult to analyze where/what went wrong.

Gurobi **computeIIS()** functionality provides an easy way to deal with this.

Simple sample output of **computeIIS():**

```
\ LP format - for model browsing. Use MPS format to capture full model detail.
Maximize

Subject To
 client_required_additional_saving_capacity_constraint_2027:
   client_required_additional_savings[2027] = 0
Bounds
 client_required_additional_savings[2027] >= 1
End
```

Q & A

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