# Proven Techniques for Solving Financial Problems with Gurobi

GUROBI OPTIMIZATION

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# Proven Techniques for Solving Financial Problems with Gurobi

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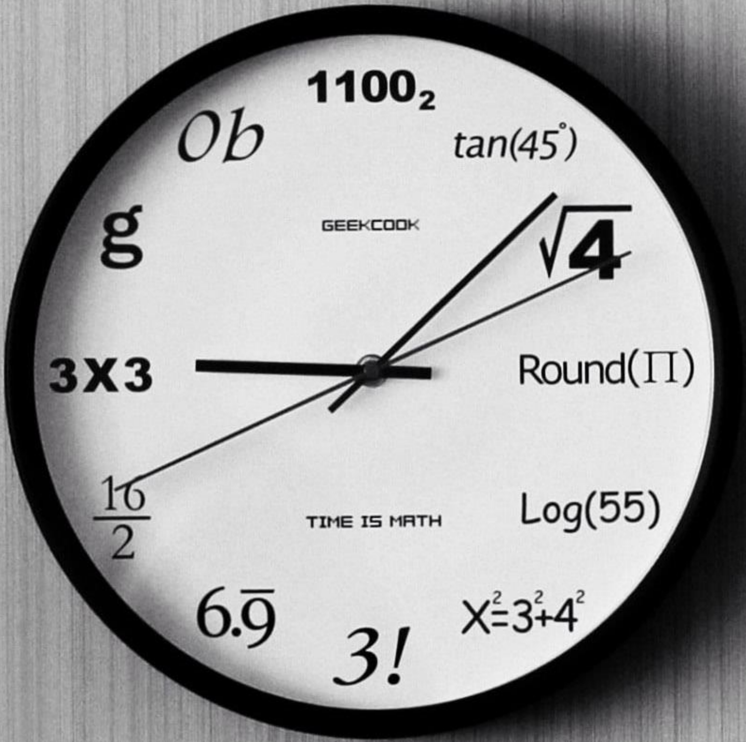


Agenda:

**1.** Introduction to portfolio optimization

**2.** Passive Investing: modeling and computational issues

**3.** Hedge of Banking ALM: alternative application of portfolio optimization

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# 01

# Introduction to Portfolio Optimization

# finor

Finance & Operations Research

**Finor** (Finance & Operations Research) is composed by **professionals with wide academic and practical experience in finance and computational-mathematical methods.** We offer modern scientific methods to help decision-making in companies operating with risk management, portfolio selection, credit scoring, pricing, cash flow optimization and others.

Because the world is multifunctional, we believe the solution to complex problems is in the interface between **technology and finance.**

# A More Technical Overview

Finor Core

| Complex Financial Calculations | AI - Machine Learning (ML) | Mathematical Optimization |
|---|---|---|
| i.e. cash flow of complex networks | i.e. credit, fraud, pricing | i.e. portfolio, ALM, index-tracking |

**MLOps and Data Engineering**

We employ several different Machine Learning methods (which can also be regarded as statistical-computational methods) such as multivariate regressions, cluster/variance analyses, neural networks, and many others.

Finor also applies various techniques for optimization (linear, mixed-integer, stochastic, heuristics) and simulation (Monte Carlo) to solve many problems related to financial modeling.

# Portfolio Optimization | Finance and OR

George Dantzig

Harry Markowitz

## Finance and OR have a long-time relationship

In 1950s, both were at RAND, and, in a OR note in 2002, Markowitz mentions the influence from Dantzig on the solution of the seminal portfolio selection problem.

> Key aspects to the practical use of portfolio optimization is on the interfaces:
> **Financial Intuition** + **OR** + **Econometrics** + **Tech** (Computing, Data Engineering...)

# Key Aspects of OR in Practical Portfolio Optimization

Our Personal View

**Model Size**: integration and development of financial markets creates thousands of options for asset allocation. Just on NYSE and NASDAQ, thousands of Stocks and ETFs are available; multiply that by all the other markets around the globe.

CVM Comissão de Valores Mobiliários FDIC (...)

**Model Constraints**: regulatory, trading, managerial policies play a major role for the decision maker. In general, they are represented by constraints in a mathematical optimization model.

# Structure of a Portfolio Optimization Problem

| | |
|---|---|
| **Maximize/Minimize - Some Objective** | $f(x)$ |
| **Subject to – Equalities/Inequalities** | $g_i(x) \leq b_i \quad i = 1, \ldots, m$ |
| **Variables – Continuous, Integer (Binary)** | $x \in \mathbb{R}^n$<br>$x_j \in \mathbb{Z}\ for\ j = 1, \ldots, p$<br>$1 \leq p < n$ |

# Structure of a Portfolio Optimization Problem

Objective Function:

**Minimize f(X)**

- Variance, VaR, Cvar, some other risk metric
- Transaction costs
- Tracking error
- Duration GAP

**Maximize f(X)**

- Long-term wealth
- Return

**IN FINANCE, in general (very often), this objective functions are non-linear (quadratic).**

# Structure of a Portfolio Optimization Problem

**Constraints (translating trading, regulatory, qualitative/managerial policies)**

- Transaction Costs: trading costs, cardinality (less asset to be managed – less operational complexity), turnover;
- Liquidity (or trading) constraints: market impact, liquidity availability;
- Managerial preferences: bounds on weight, long-only allocation, exposure (segment, country, composition/diversification);
- Regulatory constraints (i.e. pension funds): maximum allocation in an asset class, insolvency probability limit;
- Risk management: risk contribution.

**Linear, Non-linear (quadratic, non-quadratic)**
**Integer (binary) variables**

KOLM, P.; TÜTÜNCÜ, R.; FABOZZI, F. 60 Years of portfolio optimization: Practical challenges and current trends. **European Journal of Operational Research**, v. 234, n. 2, p. 356-371, 2014.

# Structure of a Portfolio Optimization Problem

Objective **Function**:

- Linear
- Non-linear (**i.e. quadratic**)

+

Translation of trading, regulatory, qualitative/managerial policies into constraints:

- **Linear**
- Non-linear (i.e. quadratic)
- **Continuous**, Integer (**binary**) variables

MIP – Mixed-Integer Programming

MINLP – Mixed-Integer Non-Linear Programming

**(i.e. MIQP – Mixed Integer Quadratic Programming)**

# Large-Scale Optimization and Algorithm Speed

Decision Variables Large-Dimensionality:

- Assets: large number of countries, asset classes and assets in each class create almost endless possibilities for asset allocation;
- Time-periods: multiple periods asset allocation models quickly increase the number of variables in a multiple-period portfolio selection model.

The Boom of Algorithm Trading:

- High-Frequency: seconds, milliseconds;
- Medium-Frequency: few minutes, some hours a day;
- Low-Frequency: few days, weeks.

→ Trading and Backtesting → **Computation Time**

# Demanding Needs for Portfolio Optimization

**Large Scale**

**+ MIP - MINLP (MIQP, MIQCP)**

**+ Fast Computing**

# Portfolio Optimization Applications and Interdisciplinarity

**Active Investing**

Mean-variance
Minimum variance
Risk Parity
...

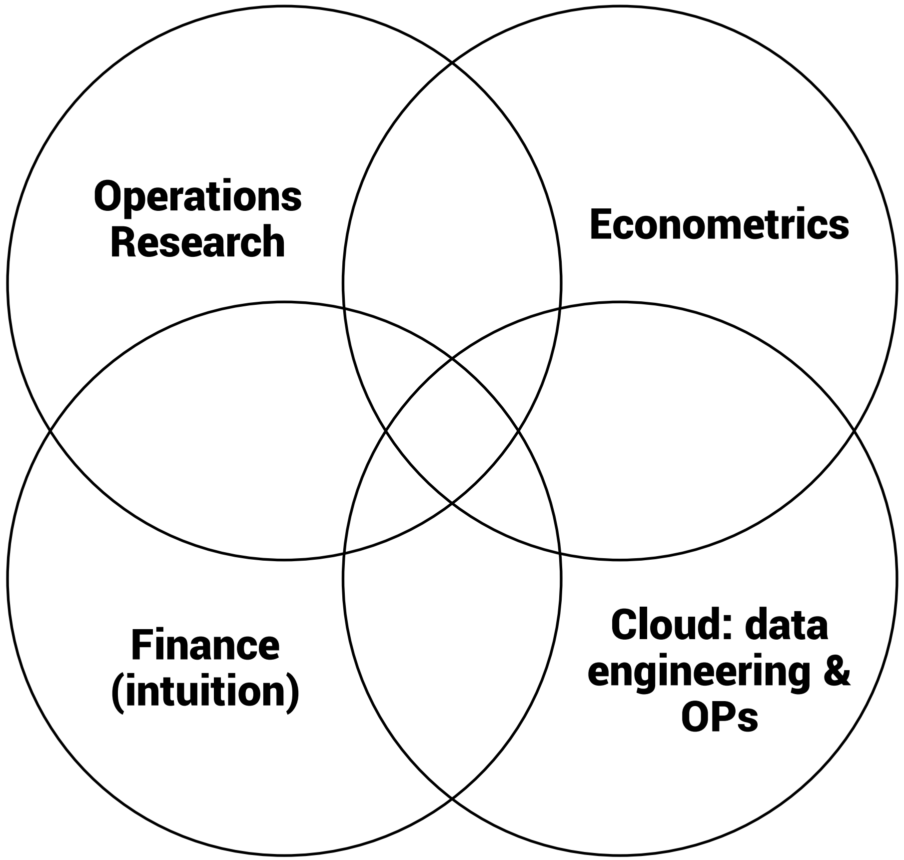
**Passive (Semi) Investing**

Index-tracking
Enhanced Index
...

**ALM**

Pension Fund (asset allocation)
Banking ALM (Hedge)

Interdisciplinary



# 02

# Passive Investing: modeling and computational issues

# Tracking Market Indices

- Passive/index investing has become very popular in the last decade
- Market indices (e.g. SP500) are composed of individual stocks
- Buying the whole index can be troublesome!

Components of the S&P 500

| # | Company | Symbol | Weight | | Price | Chg | % Chg |
|---|---|---|---|---|---|---|---|
| 1 | Apple Inc. | AAPL | 7.033773 | ▼ | 165.92 | -2.72 | (-1.61%) |
| 2 | Microsoft Corporation | MSFT | 5.952659 | ▼ | 290.48 | -4.56 | (-1.55%) |
| 3 | Amazon.com Inc. | AMZN | 3.636478 | ▼ | 3,019.20 | -46.67 | (-1.52%) |
| 4 | Alphabet Inc. Class A | GOOGL | 2.186615 | ▼ | 2,649.15 | -36.50 | (-1.36%) |
| 5 | Alphabet Inc. Class C | GOOG | 2.032445 | ▼ | 2,648.50 | -34.10 | (-1.27%) |
| 6 | Tesla Inc | TSLA | 1.929384 | ▼ | 841.61 | -18.39 | (-2.14%) |
| 7 | NVIDIA Corporation | NVDA | 1.69278 | ▼ | 232.72 | -6.77 | (-2.83%) |
| 8 | Berkshire Hathaway Inc. Class B | BRK.B | 1.535815 | ▼ | 315.32 | -3.82 | (-1.20%) |
| 9 | Meta Platforms Inc. Class A | FB | 1.414989 | ▼ | 217.30 | -2.25 | (-1.02%) |
| 10 | JPMorgan Chase & Co. | JPM | 1.208363 | ▼ | 151.40 | -2.52 | (-1.64%) |

Source: https://www.slickcharts.com/sp500

# What is Index tracking?

Index tracking is using a smaller group of stocks to replicate a market index

## Why?

Lower trading cost (buy and sell M stocks and not N)

Less assets to manage results in lower infrastructure needs

## It can become quite complex:

Regulatory issues (max weight per asset class)

Liquidity constraints (implicit and explicit costs)

Market volatility

Computational limitations

# Asset Constrained Index Tracking

$$\min \frac{1}{T}\sum_{t=1}^{T}\left(\sum_{i\in I} w_i r_{t,i} - R_t\right)^2$$

*Subject to:*

$$\sum_{i\in I} w_i = 1$$

$$w_i \geq 0 \qquad \forall i \in I$$

$$w_i \leq z_i \qquad \forall i \in I$$

$$z_i \in \{0,1\}$$

$$\sum_{i\in I} z_i \leq K$$

***Where***:

$I$ : set of available assets

$T$ : Number of time periods

$w_i$ : Weight of asset i in the tracking-portfolio

$z_i$ : Binary Variable $(0,1)$ for asset i

$R_t$ : Returns of index at time t

$r_{i,t}$ : Return of asset i at time t

$K$ : Maximum number of allowed assets

# Python and Gurobi

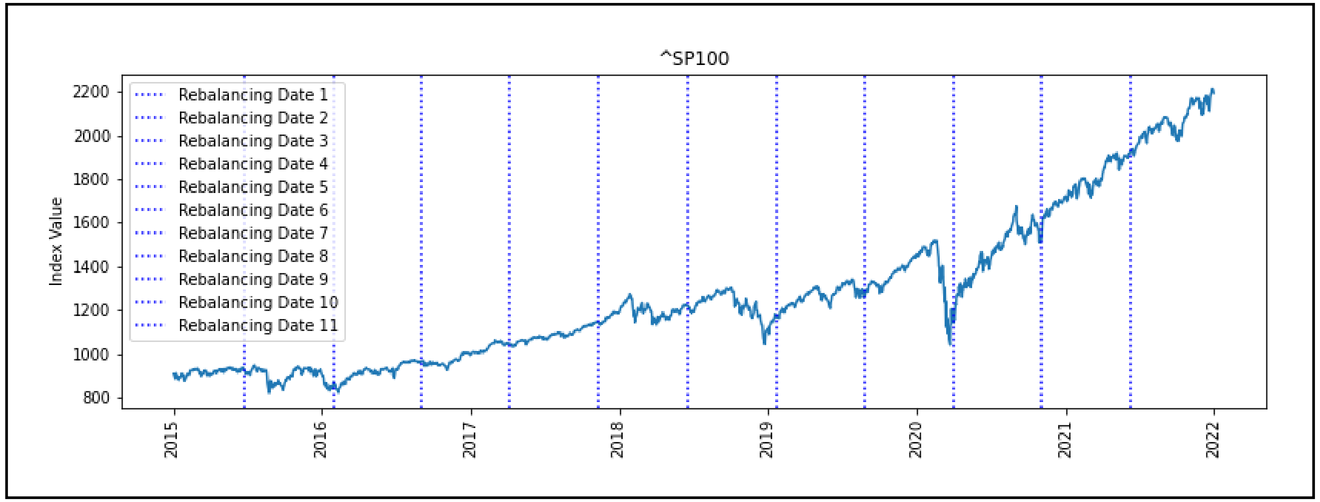
- Import Data
- Prepare Data
- Set optimization problem
- Solve it!
- Analyze results

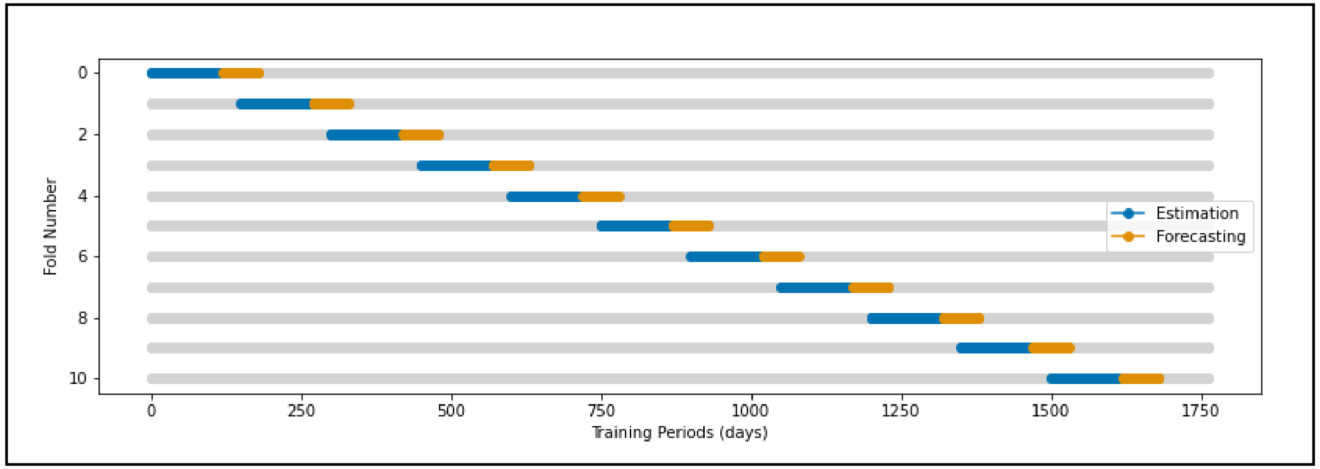
```
# Create an empty model
m_const = gp.Model('asset_contrained_model')

# PARAMETERS

K = 10 # max number of assets in portfolio

# w_i: the i_th stock gets a weight w_i
w = pd.Series(m_const.addVars(sampled_tickers,
                              lb = 0,
                              ub = 1,
                              vtype = gp.GRB.CONTINUOUS),
               index=sampled_tickers)

# [NEW] z_i: the i_th stock gets a binary z_i
z = pd.Series(m_const.addVars(sampled_tickers,
                             vtype = gp.GRB.BINARY),
                index=sampled_tickers)

# CONSTRAINTS

# sum(w_i) = 1: portfolio budget constrain (long only)
m_const.addConstr(w.sum() == 1, 'port_budget')

# [NEW]  w_i <= z_i: restrictions of values of w_i so take
for i_ticker in sampled_tickers:
    m_const.addConstr(w[i_ticker] <= z[i_ticker],
                f'dummy_restriction_{i_ticker}')
```

Full code at Colab

# A more realistic setup

- **Cross validation and rebalancing** (re-estimating) portfolios
- Within the training period, we use a window of 120 days for optimization, 30 for testing, separated by 60 days, resulting in 11 folds of data
- Within each fold, we estimate new index tracking weights using Gurobi and calculate index error over at corresponding period

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# Results (asset constrained model)

Local desktop computer could not finish execution with a solution for any K > 5 (N = 97)

But, Gurobi's results were good, despite lack of convergence

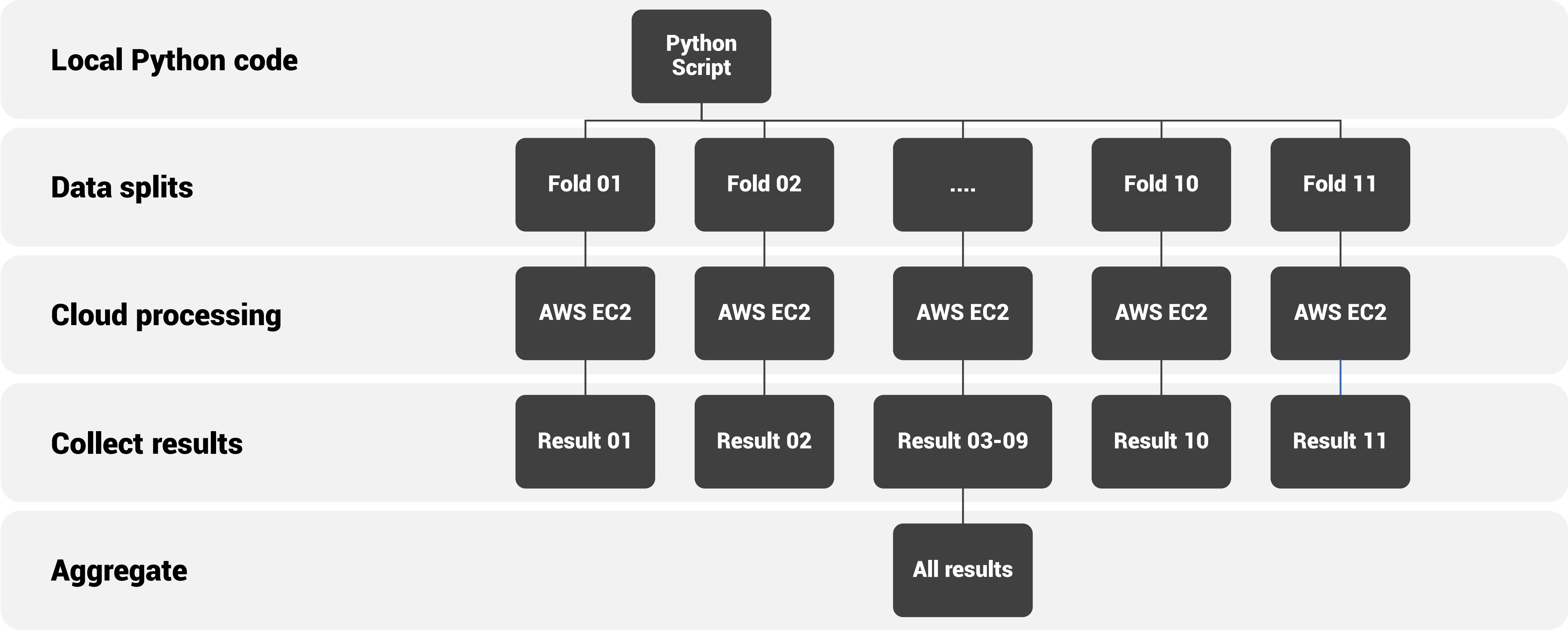
# Heuristics and cloud computing

- Heuristics to the rescue!
  - Current composition of index is publicly available:
    - **Filtering** assets by ordering index composition (the 51th asset has 0.61% of the index)
    - **Warm-start** parameters with unconstrained optimization for the K assets ranked by current weight
- Increasing computational power using AWS EC2 servers in a distributed fashion

| AWS instance type | Cost by hour (USD) | vCores \| threads used by Gurobi* | RAM (GB) |
|---|---|---|---|
| c5.4xlarge | 0.68 | 16 \| 8 | 32 |
| C5.9xlarge | 1.53 | 36 \| 18 | 72 |
| C5.18xlarge | 3.6 | 72 \| 32 | 144 |

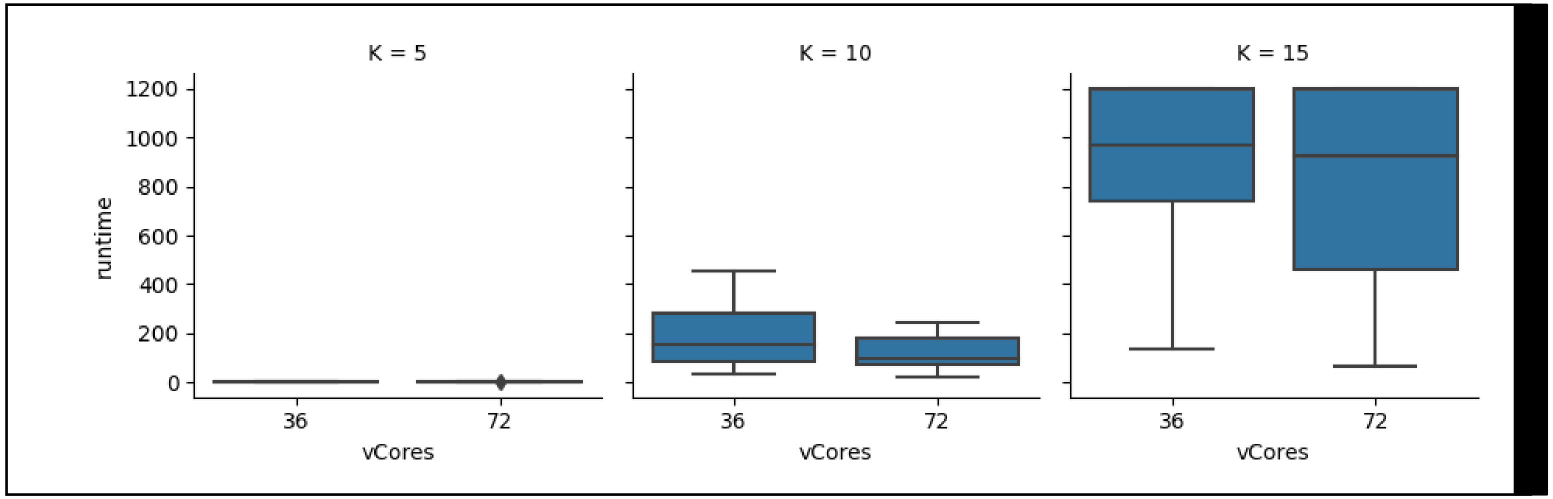
*Gurobi uses a maximum of 32 threads
Source: https://aws.amazon.com/pt/ec2/pricing/on-demand/

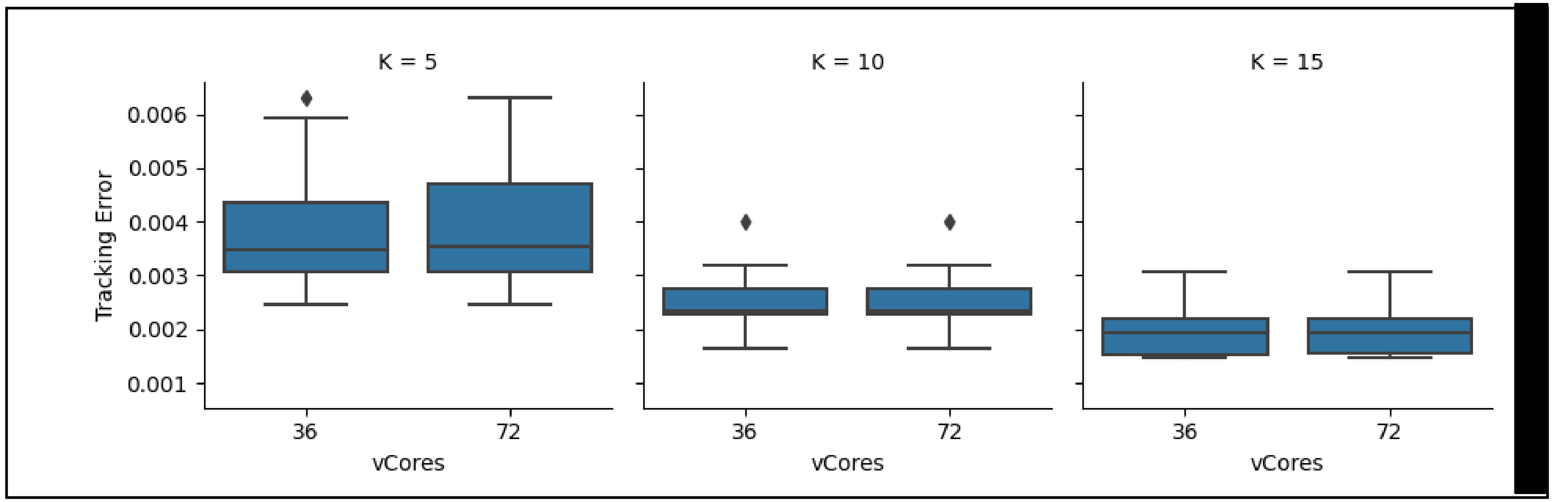
# Using Distributed Cloud



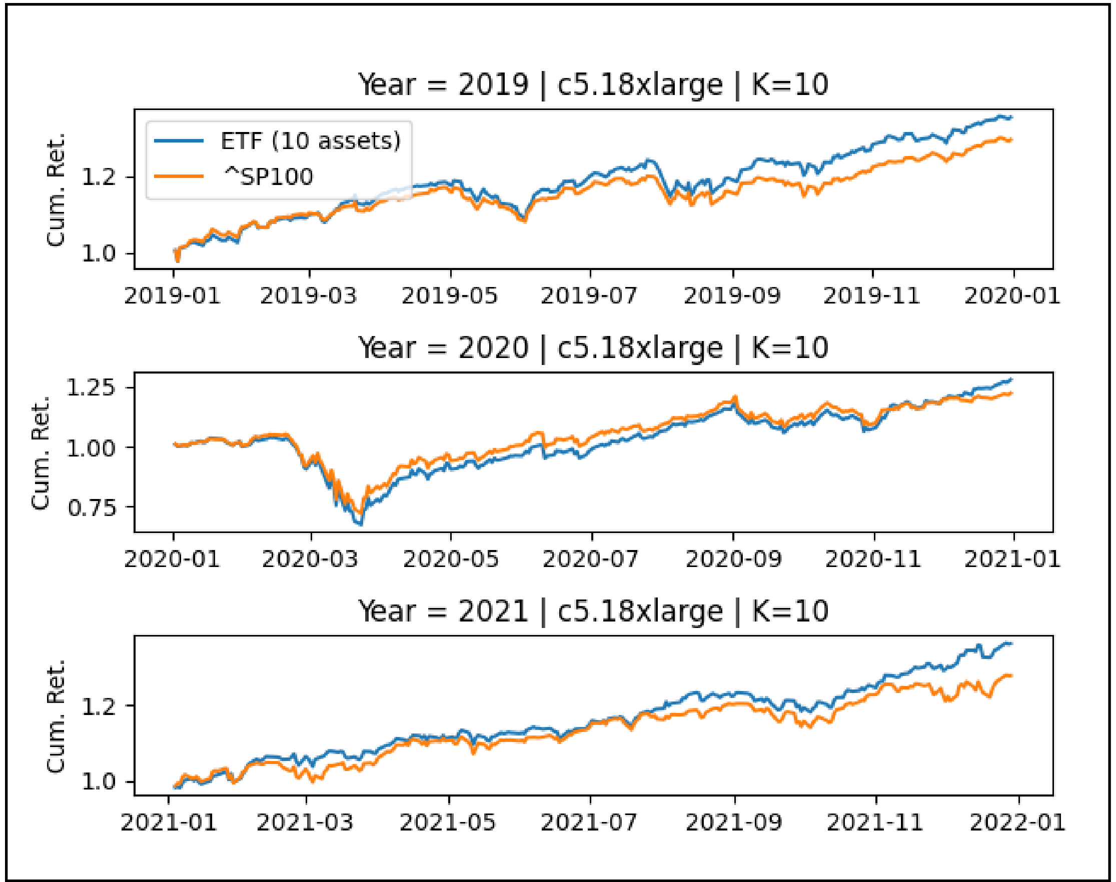

**Total execution time:** 20 minutes (4 values of K, 2 server types, 11 folds)
**Total cost:** 20.41 USD

# Results with heuristics (1)



# Cumulative Return (2019-2021)

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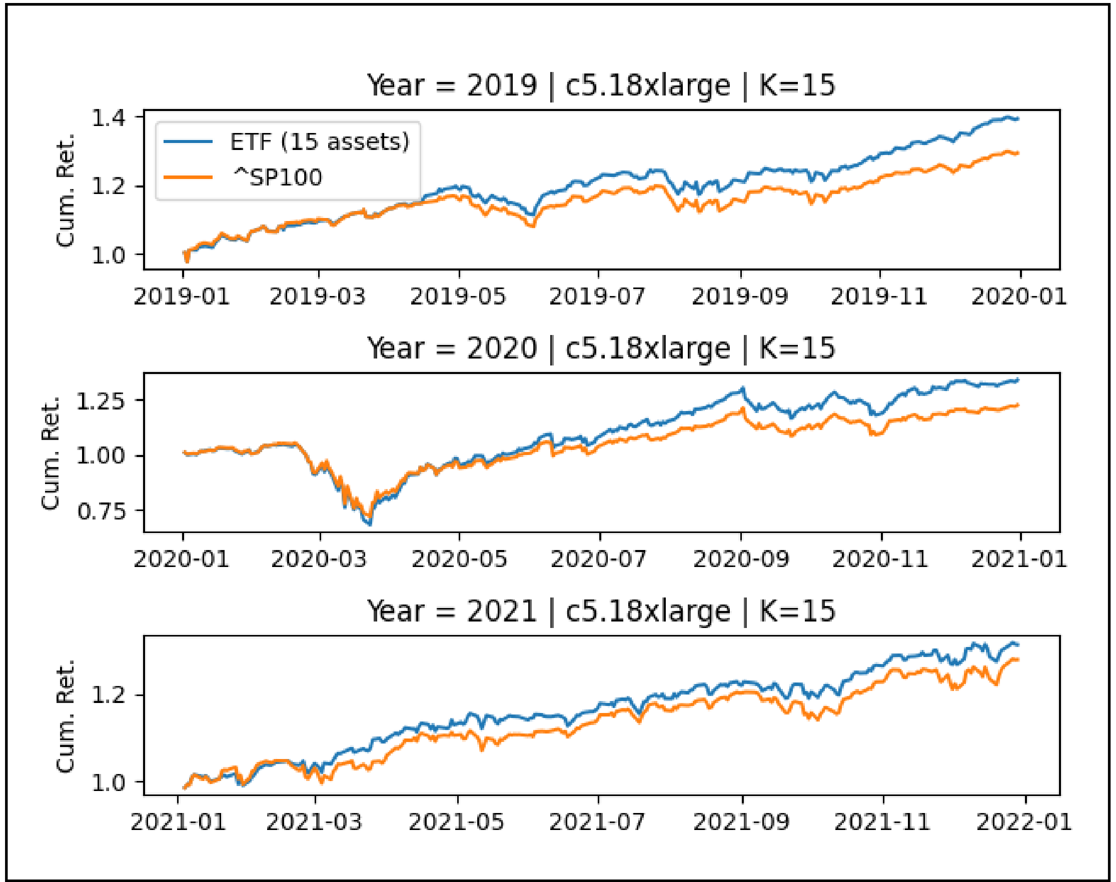


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# Conclusions

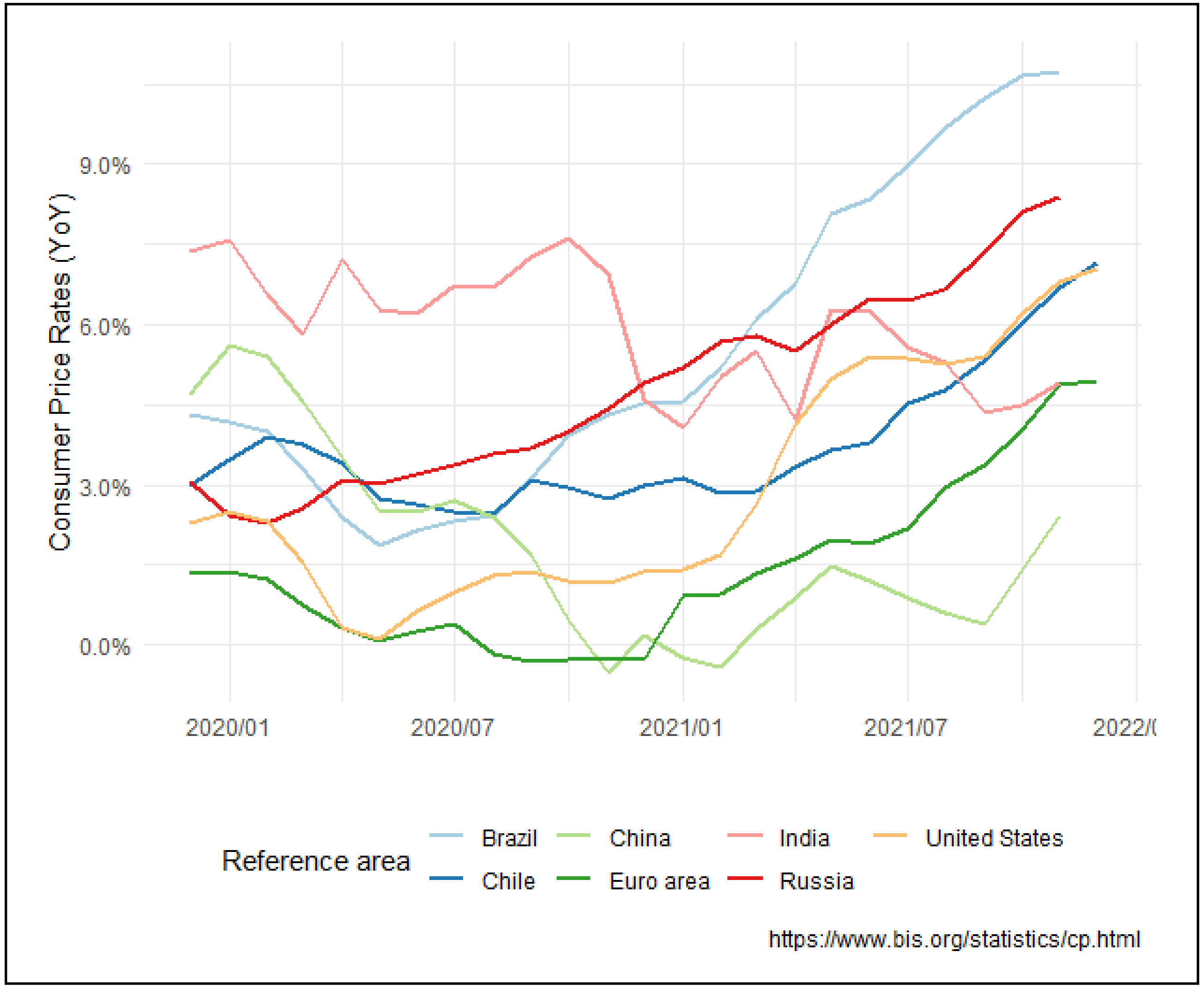
- Solving an asset constrained index tracking can be tough!
- Gurobi and parallel iterations help a lot!
- How to improve it?
  - ✓ Use cloud **batch** services with pre-configured clusters (AWS/Azure Batch)
  - ✓ **Docker** containers for a reproducible environment (however, performance should be tested)
  - ✓ **Cloud storage** for files and results (easier data retrieval and dump)
  - ✓ Monitor data and model pipelines with ML tools such as AirFlow and mlflow
  - ✓ Constant monitoring of tracking results and recalibration based on logical rules


# 03

# Hedge of Banking ALM: alternative application of portfolio optimization

# Hedging a portfolio of fixed income securities

Global inflation has been a major concern recently and central banks have acknowledged the need to change monetary policy



# Hedging a portfolio of fixed income securities

Case: Brazilian Retail Bank

**Bank Balance Sheet**

**Asset-Side**: short-medium fixed-rate loans

**Liability-Side**: really short-term floating rate funding

Fixed-rate credit portfolio: bank is long on the loan, then it is **short on the interest rate**

↑ Interest rate ↓ Credit portfolio ↑ IR Hedge

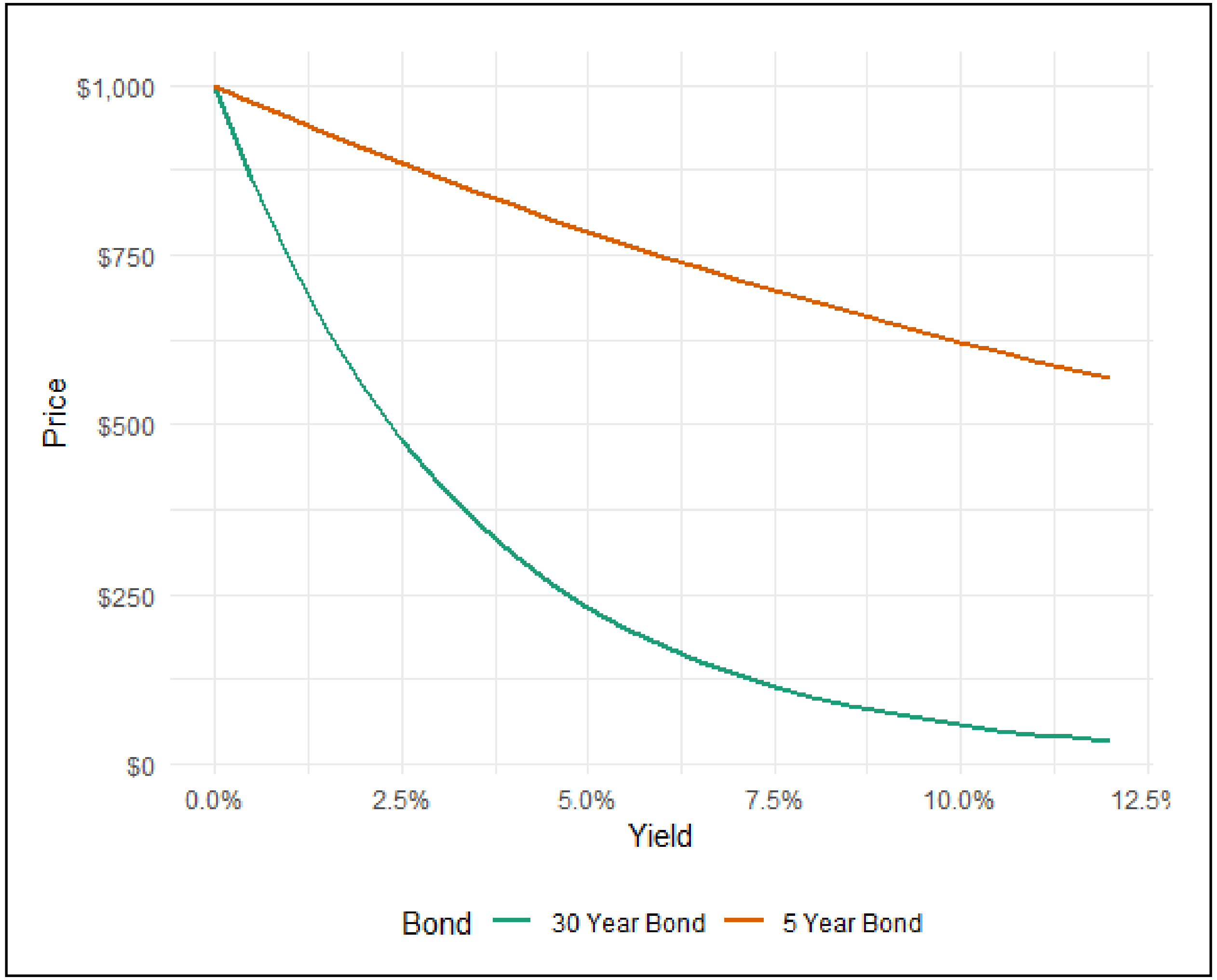
↓ Interest rate ↑ Credit portfolio ↓ IR Hedge

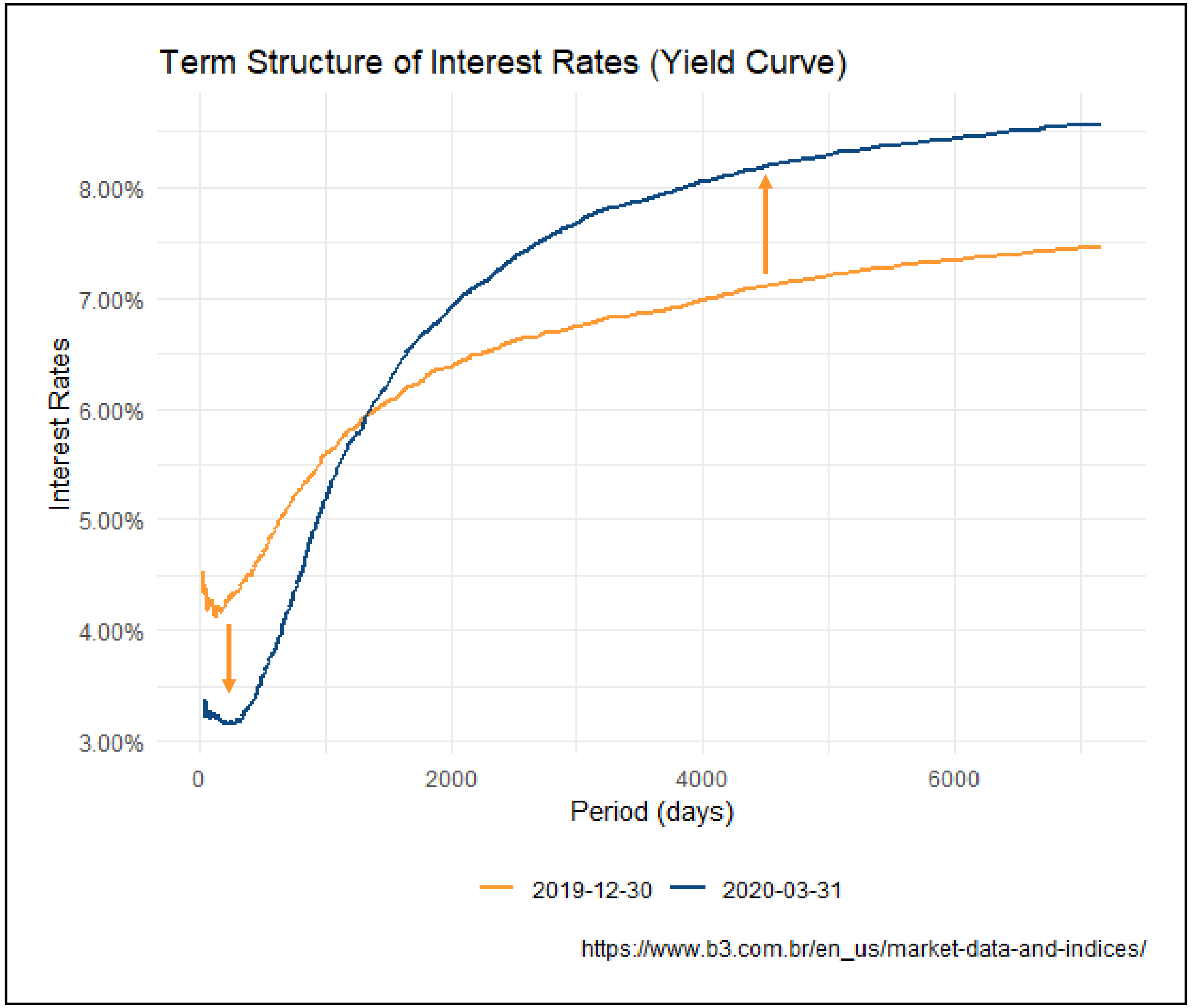
To hedge market risk using interest rate future derivatives, we need to be long in interest rates/short contract price. Then, the bank will be focused on its main business: managing **"credit risks"** (ensuring credit spreads are realized).

# Hedging a portfolio of fixed income securities
# **Simplified Example**

**The traditional way** of hedging implies in ascertaining the impact of a change of interest rates in the value of the portfolio: this sensitivity is called Duration.

**The longer the maturity** of a bond, the larger is its portfolio sensitivity.






# Hedging a portfolio of fixed income securities

**This simple hedging** with one derivative instrument only works if all interest rates along the term structure of interest rates (yield curve) show very high correlation.


# Hedging a portfolio of fixed income securities

With the characteristics of the credit portfolio we are hedging, a simple but effective approach with practical constraints is sufficient:

- Partition **Yield curve** based on key-rates (buckets);
- Calculate Dollar duration and Dollar Convexity in each bucket along the yield curve (for cash flows and hedges);
- Take into account liquidity requirements;
- Minimize costs (operational complexity).

# Hedging a portfolio of fixed income securities

## Simplified Example with Practical Constraints

Once you add many key rates along the yield curve, liquidity and operational costs come into play.

A trade desk shouldn't rely in non-liquid derivatives for their hedging, as well as using too many instruments so that your firm face a costly operational nightmare.

Finor (with Gurobi as the solver) can help your firm by creating a hedging strategy that takes into account liquidity, operation costs while maintaining optimum hedge efficiency.

# Mathematical Model

## Objective

Hedge market risks: portfolio immunization

Minimize operational costs

## Managerial Constraints

Effectiveness of the hedge

Liquidity requirements

**MIP – Mixed Integer Programming**

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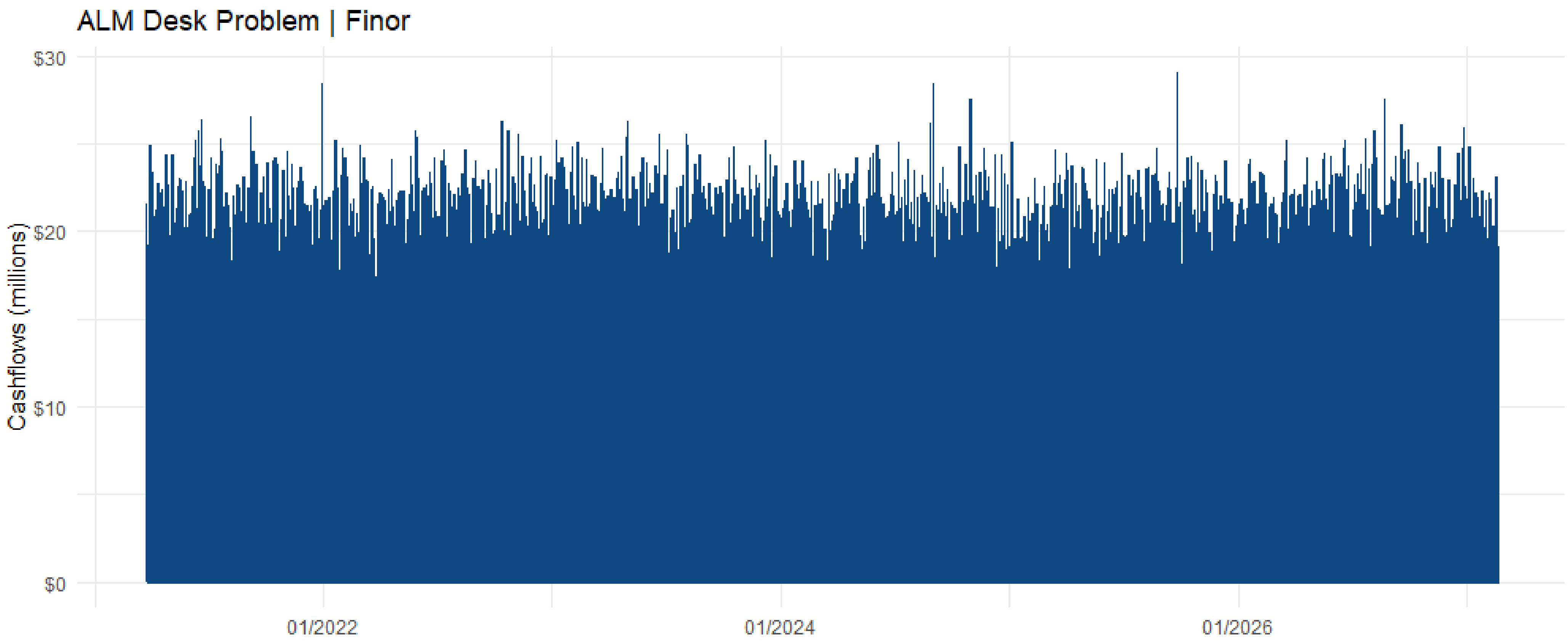


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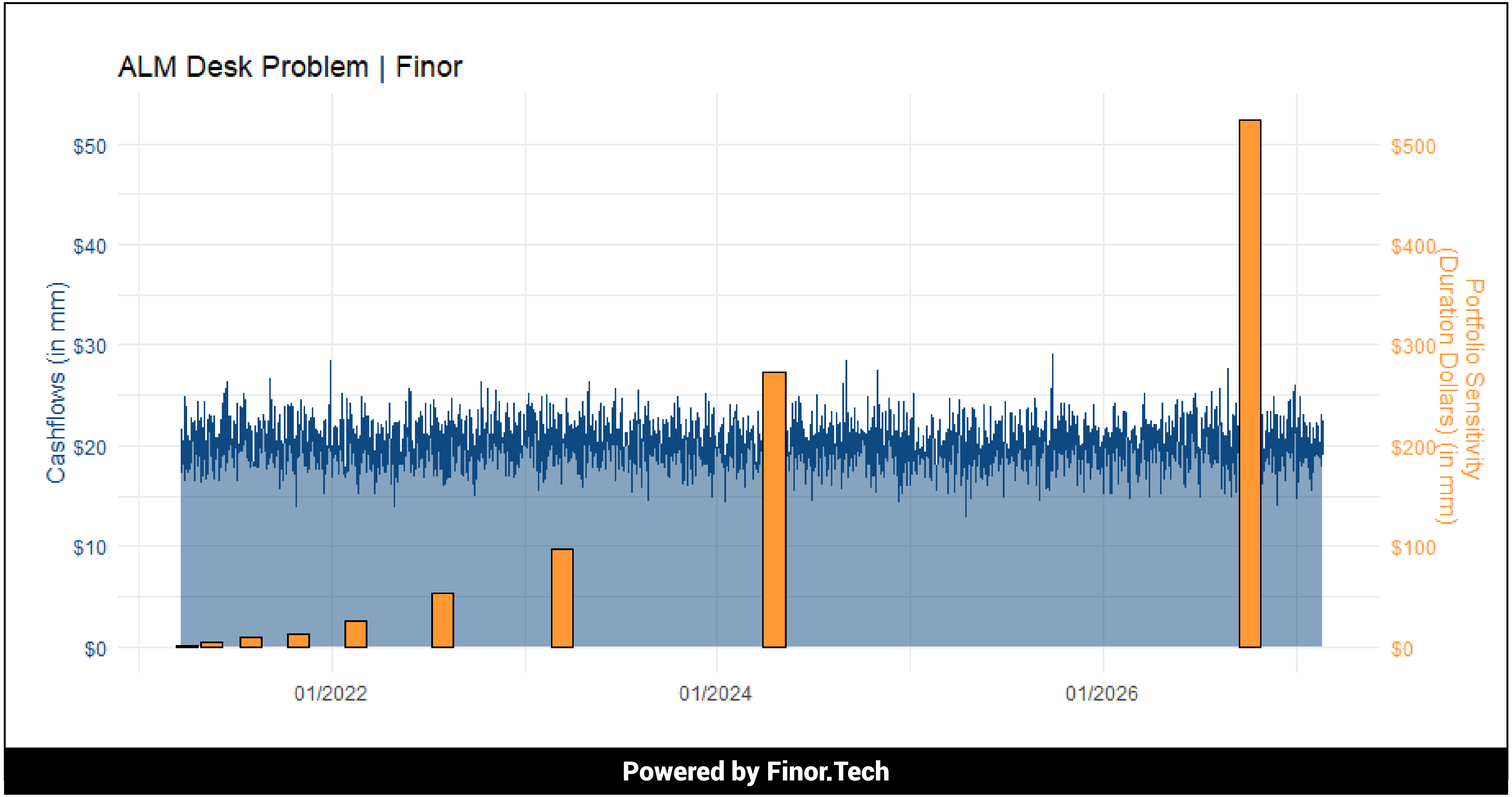
ALM Desk Problem | Finor
Cashflows (in mm)
$50
$40
$30
$20
$10
$0
Portfolio Sensitivity (Duration Dollars) (in mm)
$500
$400
$300
$200
$100
$0
01/2022
01/2024
01/2026
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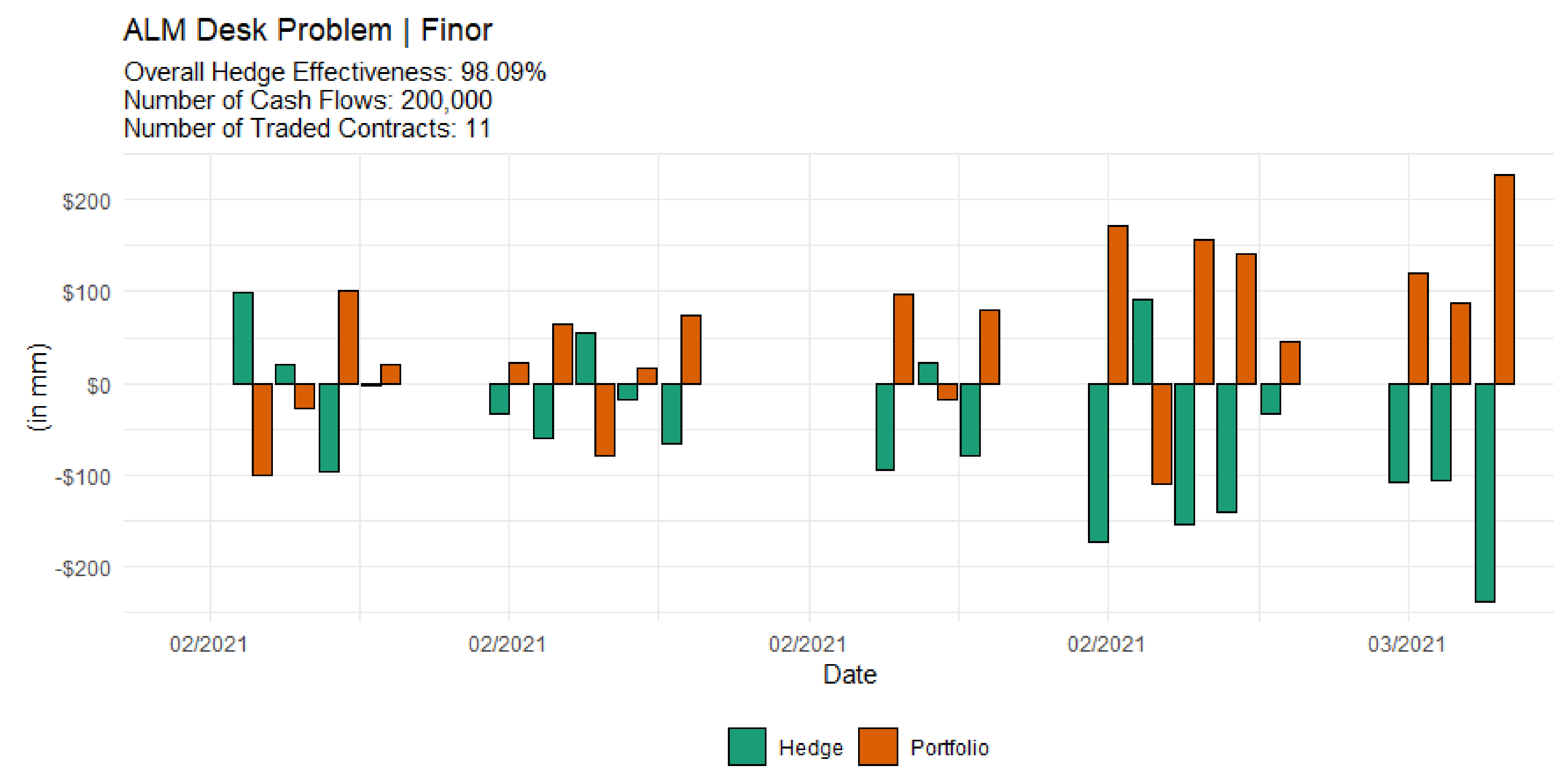
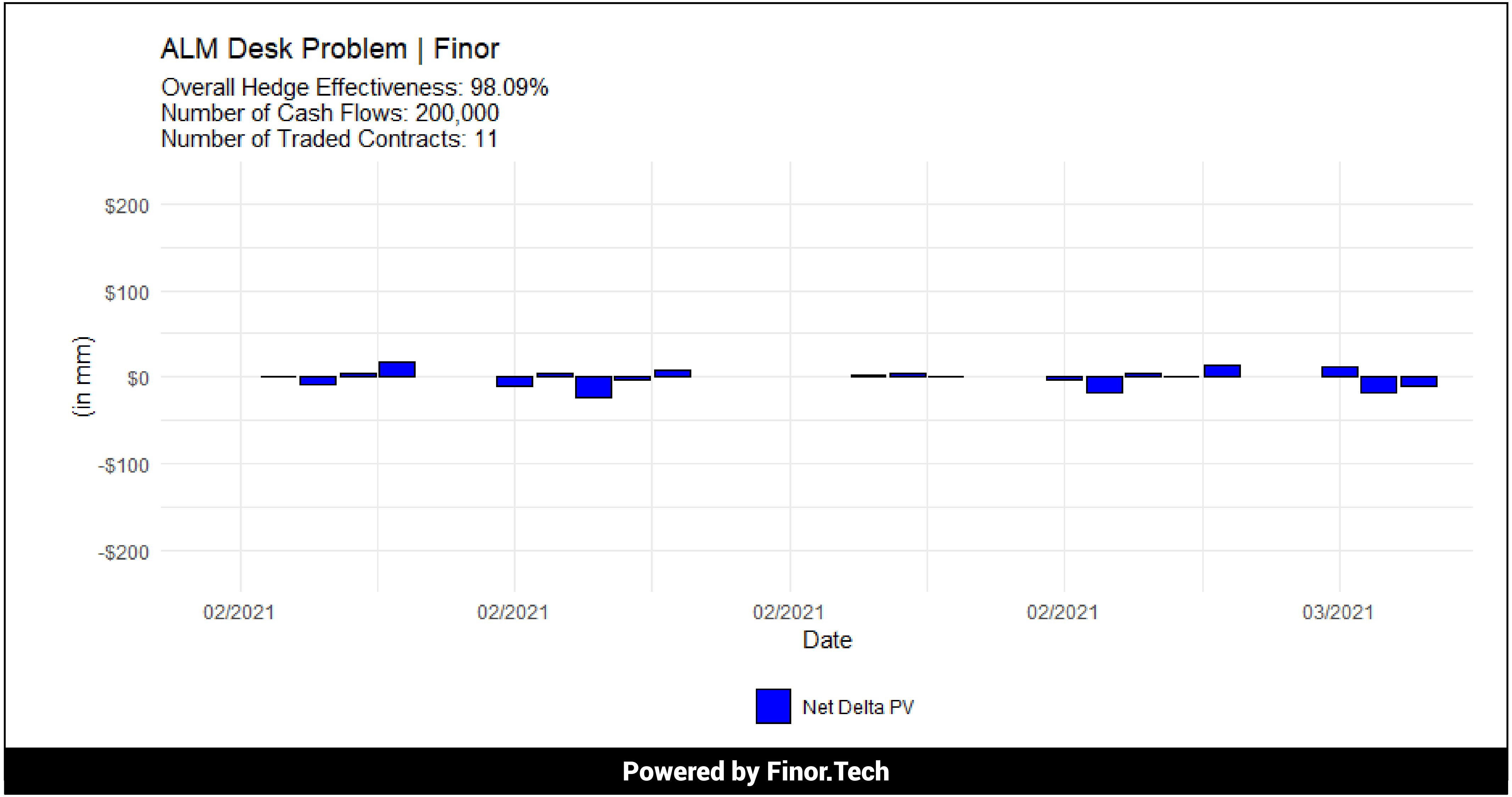
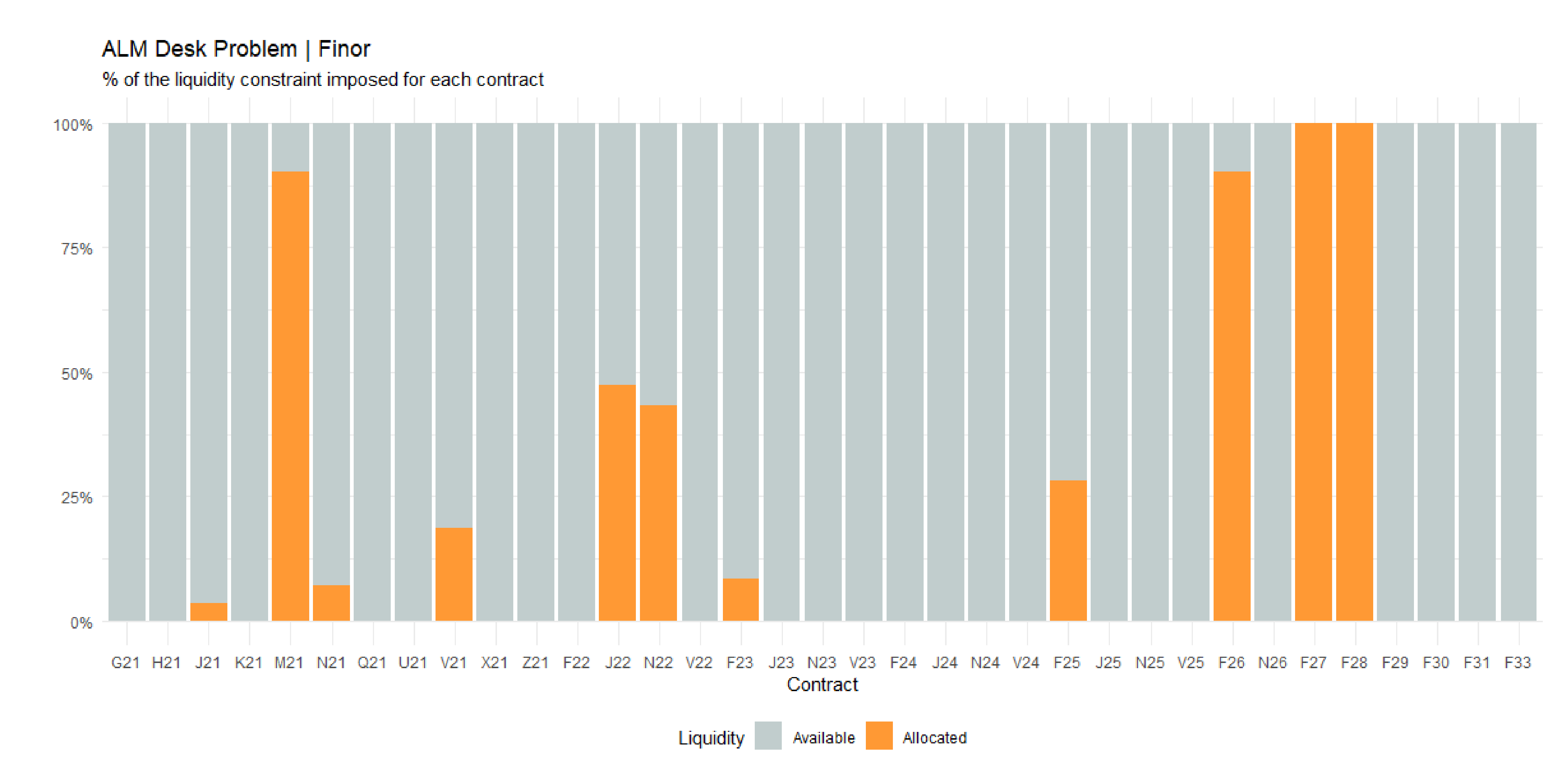


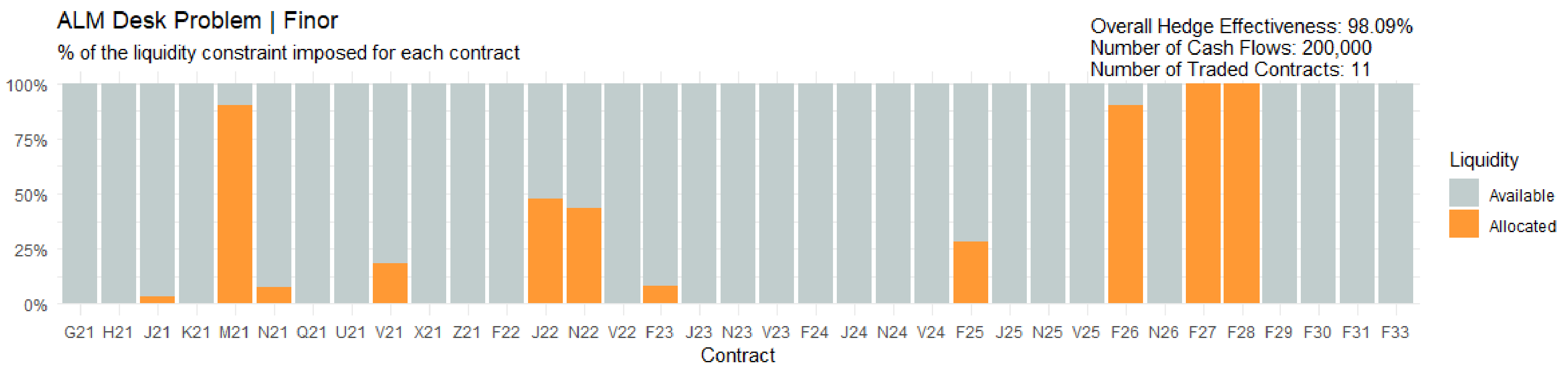

**For Hedge, the bars represent daily margin adjustments. For Portfolio, the bars represent daily PV changes.**


ALM Desk Problem | Finor
Overall Hedge Effectiveness: 98.09%
Number of Cash Flows: 200,000
Number of Traded Contracts: 11
$200
$100
$0
-$100
-$200
(in mm)
02/2021
02/2021
02/2021
02/2021
03/2021
Date
Net Delta PV
Powered by Finor.Tech

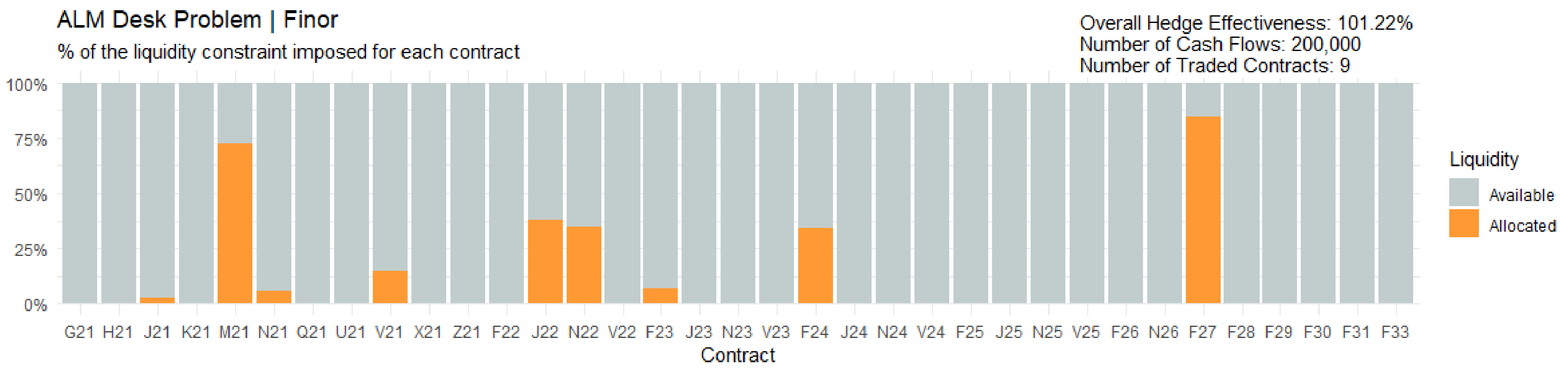



**Y-Axis means the amount of liquidity consumed from the maximum trading limits defined exogenously.**

## Liquidity Constraint = **20%** of Monthly averaged liquidity




## Liquidity Constraint = **25%** of Monthly averaged liquidity



# References

KOLM, P.; TÜTÜNCÜ, R.; FABOZZI, F. 60 Years of portfolio optimization: Practical challenges and current trends. **European Journal of Operational Research**, v. 234, n. 2, p. 356-371, 2014.

MARKOWITZ, H. Efficient portfolios, sparse matrices, and entities: a retrospective. **Operations Research**, v. 50, n. 1, p. 154-160, 2002.

## INDEX-Tracking

VIEIRA, E.; FILOMENA, T.; SANTANNA, L.; LEJEUNE, M. Liquidity-constrained index tracking optimization models. **Annals of Operations Research**, Forthcoming, 2021.

BUBICZ, E.; FILOMENA, T.; SANTANNA, L.; VIEIRA, E. Rebalancing index tracking portfolios with cumulative sum (CUSUM) control charts. **Engineering Economist**, p. 319-345, 2021.

SANTANNA, L. ; FILOMENA, T. P. ; BORENSTEIN, D. ; GUEDES, P. . Index tracking with controlled number of assets using a hybrid heuristic combining genetic algorithm and non-linear programming. **Annals of Operations Research**, v. 258, p. 449-867, 2017.

# References

## ALM

DE OLIVEIRA, A.D. ; FILOMENA, T. P. Compacting multistage stochastic programming models through a new implicit extensive framework. Journal of Computational Science, , v. 43, p. 101128, 2020. .

DE OLIVEIRA, A. D.; FILOMENA, T. P.; PERLIN, M.; LEJEUNE, M.; MACEDO, G. A. Multistage Stochastic Programming Asset-Liability Management Model - An Application to the Brazilian Pension Fund Industry. Optimization and Engineering, v. 18, p. 349-368, 2017.

# Thank you.

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